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Dynamical analysis of plant disease models with cultural control strategies and economic thresholds

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Abstract

In this paper plant disease models including impulsive cultural control strategies were developed and analyzed. The sufficient conditions under which the infected plant free periodic solution with fixed moments is globally stable are obtained. For the model with an economic threshold (ET) of infected plants, detailed investigations imply that the number of healthy plants either goes to extinction or tends to infinity, and the maximum value of infected plants is always less than the given ET. In order to prevent the healthy plant population going to extinction, we further propose a bi-threshold-value model, which has richer dynamical behavior including order 1-k or order k-1 periodic solutions with $k \geq 1$. Under certain parameter spaces, the infected plant free periodic solution is globally stable for the bi-threshold-value model. The modeling methods and analytical analysis presented can serve as an integrating measure to identify, evaluate and design appropriate plant disease control strategies.

Keywords: Plant disease; Epidemiology; Cultural control; Economic threshold; Bi-threshold value

1. Introduction

Plant viruses are an important constraint to crop production worldwide, and cause serious losses in yield and quality of cultivated plants. Several plant diseases caused by plant viruses in cassava (*Manihot esculenta*), sweet potato (*Ipomoea batatas*) and plantain (*Musa* spp.) are among the main constraints to sustainable production of these vegetatively propagated staple food crops in lesser-developed countries [7,8,24]. Therefore farmers have been evolving practices for combating the various plagues suffered by crops, and growing understanding of the interactions between pathogen and host has enabled us to develop a wide array of measures for the control of specific plant diseases.

Such experiences have led to the development of integrated management concepts for virus diseases that combine available host resistance, cultural, chemical and biological control measures. Examples of how epidemiological

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information can be used to develop effective integrated disease management (IDM) strategies for diverse situations have been described [3,12–14]. IDM involves the selection and application of a wide range of control strategies that minimize losses and maximize returns. A cultural control strategy including replanting, and/or removing (roguing) diseased plants is a widely accepted treatment for plant epidemics which involves periodic inspections followed by removal of the detected infected plants [5,6,11,28].

Mathematical models of plant-virus disease epidemics were developed to provide a detailed exposition on how to describe, analyze, and predict epidemics of plant disease for the ultimate purposes of developing and testing control strategies and tactics for crop protection [25,2,5,9]. A simple model for plant disease with a continuous cultural control strategy, such as replanting and roguing or removing, is as follows:

$$\begin{cases} \dfrac{\mathrm{d}S(t)}{\mathrm{d}t} = \sigma\phi - \beta S(t)I(t) - \eta S(t), \\ \dfrac{\mathrm{d}I(t)}{\mathrm{d}t} = \sigma(1-\phi) + \beta S(t)I(t) - (\eta+\omega)I(t), \end{cases} \tag{1}$$

where S, I denote the number of susceptible and infected plants, respectively. β is the transmission rate, η either denotes harvest time or the end of reproductive lifetime of plants, σ represents the total rate at which plants enter the system with a proportion ϕ for the susceptible plants and $(1-\phi)$ for infected plants, ω is the roguing (or removal) rate for the infected plants. The detailed biological background and model development can be found in [25,2]. Further, a model for the temporal spread of an epidemic in a closed plant population with periodic removals of infected plants has been considered by Fishman et al. [5] with an application to the spread of citrus tristeza virus disease. Their model helped in evaluating policies of controlling the disease and could be also modified to simulate other plant epidemics with periodic treatments.

Assuming that the increase in incidence of infection during time T is proportional to the proportion of plants already infected and to the proportion of plants still left for infection, Fishman et al. [5] proposed two types of model with Logistic growth and periodic removing of infected trees, aimed at eradicating them. Therefore, one of the main purposes of the models described in this paper is to extend the deterministic model (1), constructed for a closed plant population with periodic replanting and roguing controlling strategy, and analyze its dynamical behavior by using the theories of impulsive differential equations. The results imply that the infected plants can be completely eradicated if the period of control strategy application is sufficiently large, i.e. the infected plant free periodic solution is globally stable.

IDM involves the selection and application of a harmonious range of disease control strategies that minimize losses and maximize returns. The objective of integrated control programs is to achieve a level of disease control that is acceptable in economic terms to farmers while causing minimal disturbance to the environments of non target individuals. Note that complete eradication of the infected plants is generally not possible, nor is it biologically or economically desirable. The use of cultural controls, such as replantation and removal (roguing) of diseased plants, has been practised and can limit disease rather than eradicate it completely [2,5,9]. Therefore, a good plant disease control program should reduce the infected plants to levels acceptable to the public. This implies that there is an economic threshold (ET) above which the financial damage is sufficient to justify using such measures [22,23]. Whether or not a particular strategy for control of plant viruses is implemented by farmers depends on a wide range of socio-economic and cultural drivers [24] in addition to the effectiveness of control.

The other main purpose of this paper is to extend the model with a periodic control strategy to one model with an ET for the infected plants. That is, we formulate a state-dependent impulsive differential equation which represents situations when a cultural control strategy needs to be applied only when the infected plant population reaches the ET. For the proposed model with ET of the infected plants, we prove that if the model does not have any periodic solutions, the solutions will approach zero or the number of healthy plants tends to infinity and the number of infected plants is no larger than the given ET; if there exists a periodic solution, the number of healthy plants either goes to extinction or tends to infinity which depends on the initial values. Nevertheless, in any case the maximum value of infected plants is always less than the given ET. This also clarifies that the periodic solution is unstable. Further in order to prevent the number of healthy plants going to extinction, we propose a model with two different ETs for the healthy plants and infected plants, respectively. The dynamical behavior of this model is richer and order 1-k or order k-1 periodic solutions with $k \geq 1$ were obtained for it. Under some conditions, the infected plant free periodic solution is globally stable for the bi-threshold-value model.