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Application of mesh-adaptation for pollutant transport by water flow

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Abstract

An adaptive finite volume method is proposed for the numerical solution of pollutant transport by water flows. The shallow water equations with eddy viscosity, bottom friction forces and wind shear stresses are used for modelling the water flow whereas, a transport-diffusion equation is used for modelling the advection and dispersion of pollutant concentration. The adaptive finite volume method uses simple centred-type discretization for the source terms, can handle complex topography using unstructured grids and satisfies the conservation property. The adaptation criteria are based on monitoring the pollutant concentration in the computational domain during its dispersion process. The emphasis in this paper is on the application of the proposed method for numerical simulation of pollution dispersion in the Strait of Gibraltar. Results are presented using different tidal conditions and wind-induced flow fields in the Strait.

Keywords: Water flow; Pollutant transport; Adaptive method; Finite volume method; Unstructured grids

1. Introduction

During the last decades partial differential equations have been used as practical tools to model many environmental problems from real life. Their have also been used to approximate and predict the dynamics of such problems. The goal of the present work is to provide a simple and practical numerical model able to resolve and correctly capture the transport and dispersion of a pollutant by water flows. The underlying equations describe the free-surface flow and species equations. Here, only water flow and pollutant concentration are coupled and neither chemical reactions nor heat transfer are considered. The flow is governed by the depth-averaged Navier–Stokes equations involving several assumptions including (i) the domain is shallow enough to ignore the vertical effects, (ii) the pressure is hydrostatic, and (iii) viscous dissipation of energy is ignored. A convection–diffusion equation is used to model the distribution of pollutant concentration on the water free-surface.

The emphasis of the present work is on the application of the developed methods to a pollution dispersion in the Strait of Gibraltar. The Strait of Gibraltar has been subject of several scientific investigations ranging from meteological studies to geological researches. Its strategical location makes these investigations indispensable for understanding the global oceanography and the climate prediction. The Strait is heavily used by shipping and oil transport, being one

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of the most chronically polluted regions [6]. The most extensively studied field to date is the mean flow through the Strait of Gibraltar. There are many published material concerning for example, tides and tidal waves in the Strait, water exchange between Atlantic and Mediterranean, and salinity transfer. Almost all these studies use the shallow water equations in the formulation of their models, we refer to [1,8,7,9,11] among others. Recently attention has been shifted to the mathematical and numerical study of pollutant transport in the Strait of Gibraltar, and many models are still under validation since no experimental data is currently available. It is worth to mention that, a pollution event in the Strait can affect important ecological region containing coral reefs, species of fishes and birds, and supports crucial fishing industry. The cleaning process may require many months, human and financial resources.

To approximate numerical solution to the governing equations we implement an adaptive finite volume solver recently developed in [10,2]. The main advantages of the proposed finite volume method are (i) the implementation on unstructured meshes allowing for local mesh refinement during the simulation process, (ii) the simultaneous advection in time of the water flow and the pollutant concentration, solving both problems at the same time and with the same accuracy (iii) the ability to handle calculations of slowly varying flows or concentrations as well as rapidly varying flows containing also shocks or discontinuities, and (iv) the capability to satisfy the exact C-property and to guarantee positive values of both, water level and pollutant concentration in the transient simulations. In the computations presented in this paper we have used the concentration of pollutant as a monitoring function for mesh refinements. Results presented in this paper demonstrate high resolution of the proposed method and confirm its capability to provide accurate and efficient simulations for pollutant transport by water flows including complex topography and friction forces on unstructured grids. In Addition, the results show good behavior of the pollutant in the Strait of Gibraltar and also illustrate correct flow structures near the pollutant for different wind directions. The computational cost is much lower than implementing the finite volume on fixed meshes.

2. Equations for pollutant transport by water flow

Assuming a hydrostatic pressure and a vertically uniform horizontal velocity field, the free-surface water flow can be correctly modelled by the well-established shallow water equations. To model the transport of a pollution we consider an advection–diffusion equations for the pollutant concentration. Written in a conservative form, these equations read

$$\partial_t \mathbf{W} + \partial_x(\mathbf{F}(\mathbf{W}) - \tilde{\mathbf{F}}(\mathbf{W})) + \partial_y(\mathbf{G}(\mathbf{W}) - \tilde{\mathbf{G}}(\mathbf{W})) = \mathbf{S}(\mathbf{W}), \tag{1}$$

where $\mathbf{W}$ and $\mathbf{S}$ are the vectors of conserved variables and source term, $\mathbf{F}$ and $\mathbf{G}$ are the advective tensor fluxes, $\tilde{\mathbf{F}}$ and $\tilde{\mathbf{G}}$ are the diffusion tensor fluxes

$$\mathbf{W} = \begin{pmatrix} h \\ hu \\ hv \\ h\phi \end{pmatrix}, \qquad \mathbf{S}(\mathbf{W}) = \begin{pmatrix} 0 \\ -gh\partial_x Z - fhv - \dfrac{\tau_{bx}}{\rho} + \dfrac{\tau_{wx}}{\rho} \\ -gh\partial_y Z + fhu - \dfrac{\tau_{by}}{\rho} + \dfrac{\tau_{wy}}{\rho} \\ hQ \end{pmatrix},$$

$$\mathbf{F}(\mathbf{W}) = \begin{pmatrix} hu \\ hu^2 + \dfrac{1}{2}gh^2 \\ huv \\ hu\phi \end{pmatrix}, \qquad \mathbf{G}(\mathbf{W}) = \begin{pmatrix} hv \\ huv \\ hv^2 + \dfrac{1}{2}gh^2 \\ hv\phi \end{pmatrix},$$

$$\tilde{\mathbf{F}}(\mathbf{W}) = \begin{pmatrix} 0 \\ \nu\partial_x(hu) \\ \nu\partial_x(hv) \\ d_x\partial_x(h\phi) \end{pmatrix}, \qquad \tilde{\mathbf{G}}(\mathbf{W}) = \begin{pmatrix} 0 \\ \nu\partial_y(hu) \\ \nu\partial_y(hv) \\ d_y\partial_y(h\phi) \end{pmatrix}.$$

where u and v are the depth-averaged water velocity in x- and y-direction, h the water depth, Z the bed elevation, g the gravitational acceleration, ρ the water density, f the Coriolis parameter defined by $f = 2\omega \sin \Phi$, with $\omega = 0.000073\,\mathrm{rad\,s^{-1}}$ is the angular velocity of the earth and Φ is the geographic latitude, ν the kinematic viscosity, τ_{bx}