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Sliding mode control for polytopic differential inclusion systems

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Abstract

The stabilization problem of polytopic differential inclusion (PDI) systems is investigated by using sliding mode control. Sliding surface is designed and sufficient conditions for asymptotic stability of sliding mode dynamics are derived. A novel feedback law is established to make the state of system reach the sliding surface in a finite time. Finally, an example is given to illustrate the validity of the proposed design.

Keywords: Polytopic differential inclusion systems; Sliding mode control; Asymptotic stability

1. Introduction

At the past 50 years, the theory of linear systems have been widely and deeply investigated. Since Rudolph E. Kalman proposed the state space method of linear systems, a lot of textbooks have been published, especially, [4] and [21] are two classical works. Recently, [1] gives a more present description of linear system theory. The main target of the paper is to extend some results for linear systems to the systems described by differential inclusions (DI). Recently, the study of DI systems has been paid much attention by many authors. The DI systems are considered a generalization of the system of differential equations. Many practical systems should be described by DI systems [3,5–20]. In [3], the asymptotic stability of linear differential inclusion (LDI) systems is investigated, especially, some specific families of LDIs, such as polytopic LDIs, norm-bound LDIs, diagonal norm-bound LDIs, are studied extensively. In [18], the control Lyapunov function approach is employed to solve the stabilizing problem for single-input PDI systems. In [9], a nonlinear control design method for LDIs is presented by using quadratic Lyapunov functions of their convex hull. In [14], a frequency-domain approach is proposed to analyze the globally asymptotic stability of DI systems with discrete and distributed time-delays. In [5], the problem of tracking control of nonlinear uncertain dynamical systems described by DIs is studied. In [10], a necessary and sufficient condition for the stability of polytopic LDIs is derived by bilinear matrix equations. As we know, sliding mode control is a very effective approach for the control of nonlinear systems. It has many attractive features such as fast response, good transient response and insensitivity to variations [6,11,19]. This paper will apply sliding mode control to the stabilization of a multi-input PDI system which is more general such that the systems dealt with in [12,13,16,20] can be regarded as especial forms of the system. A novel feedback law is

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given to make the closed-loop system stable asymptotically and to reduce the chattering phenomenon. Finally, we give a simulation result to show the advantages of this method.

2. Problem formulation

Consider the following PDI system:

$$\begin{cases} \dot{\bar{x}}_1(t) = A_{11}\bar{x}_1(t) + A_{12}\bar{x}_2(t), \\ \dot{\bar{x}}_2(t) \in co\{f_i(x) + g_i(x)B(u(t) + w(t)), i = 1, 2, \ldots, N\}. \end{cases} \tag{1}$$

where $\bar{x}_1 = [x_1, x_2, \ldots, x_m]^T$, $\bar{x}_2 = [x_{m+1}, \ldots, x_n]^T$ and $x = [\bar{x}_1^T, \bar{x}_2^T]^T \in R^n$ is the system state, $co\{\cdot\}$ denotes the convex hull of a set, $f_i(x) \in R^{n-m}$, $g_i(x) \in R$, are smooth vector-valued function and function, respectively. $u(t) \in R^{n-m}$ is the control input, $B \in R^{(n-m)\times(n-m)}$ is nonsingular. A_{11}, A_{12} are matrices with compatible dimensions and (A_{11}, A_{12}) is assumed to be completely controllable. $w(t)$ is a bounded disturbance, i.e., $\|w(t)\| \leq \gamma$, with a positive constant γ. This paper assumes that $g_i(x) > 0$, for all $i = 1, 2, \ldots, N$.

By the conclusion established in convex analysis theory [15], this PDI system (1) is equivalent to the following uncertain system:

$$\begin{cases} \dot{\bar{x}}_1(t) = A_{11}\bar{x}_1(t) + A_{12}\bar{x}_2(t), \\ \dot{\bar{x}}_2(t) = \sum_{i=1}^{N} \alpha_i [f_i(x) + g_i(x)B(u(t) + w(t))]. \end{cases} \tag{2}$$

where α_i, $i = 1, 2, \ldots, N$, are uncertain parameters with the properties that $\alpha_i \geq 0$ and $\sum_{i=1}^{N} \alpha_i = 1$.

3. Main results

The control law of the uncertain system (2) is designed by two phases. Firstly, an sliding surface is chosen, and secondly, a feedback law is designed such that the state of system (2) converges to the sliding surface in a finite time. The design is stated in the following two steps.

Step 1. (A_{11}, A_{12}) is controllable, hence we can find a matrix $C_1 \in R^{(n-m)\times m}$ such that $A_{11} - A_{12}C_1$ is a Hurwitz matrix. Let $C = [C_1 I]$, the sliding surface is defined by

$$s(t) = Cx(t). \tag{3}$$

Let $s(t) = 0$. Then from (3), we have

$$\bar{x}_2(t) = -C_1\bar{x}_1(t). \tag{4}$$

Substituting Eq. (4) into the first equation of (2), the sliding mode dynamics is

$$\dot{\bar{x}}_1(t) = (A_{11} - A_{12}C_1)\bar{x}_1(t). \tag{5}$$

$A_{11} - A_{12}C_1$ is a Hurwitz matrix, hence $\bar{x}_1(t) \to 0$ as $t \to \infty$. It follows $\bar{x}_2(t) \to 0$ by (4). Thus we conclude that the sliding mode dynamics (5) is asymptotically stable.

Step 2. We present a novel feedback law in this step to drive the state to the sliding surface $s(t) = 0$ in a finite time.

The novel feedback law is considered as

$$u(t) = -\frac{B^{-1}}{a(x)} \left[C_1A_{11}\bar{x}_1(t) + C_1A_{12}\bar{x}_2(t) + k(x)\frac{s(t)}{\|s(t)\|} \right], \tag{6}$$