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Continuous and impulsive vaccination of SEIR epidemic models with saturation incidence rates☆

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Abstract

In this paper, two delayed SEIR epidemic models with continuous and impulsive vaccination and saturating incidence are investigated. The dynamical behaviors of the disease are analyzed. For continuous vaccination, we obtain a basic reproductive number R_1 and prove that if $R_1 \leq 1$ then the disease-free equilibrium is globally attractive and if $R_1 > 1$ then the disease is permanent by using the Lyapunov functional method. For impulsive vaccination, we obtain two thresholds R^* and R_* and prove that if $R^* < 1$ then the disease-free periodic solution is globally attractive and if $R_* > 1$ then the disease is permanent by using the comparison theorem of impulsive differential equation and the Lyapunov functional method. Lastly, we compared the effects of two vaccination strategies.

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Keywords: Continuous vaccination; Impulsive vaccination; Saturation incidence; Global attractivity; Permanence

1. Introduction

The study of vaccination, treatment, and associated behavioral changes related to disease transmission has been the subject of intense theoretical analysis [11,12]. Some diseases may be transferred through some direct or indirect contact with (for example, through disease vectors such as mosquitoes or other biting insects) infected individuals, this is called the horizontal transmission. Vertical transmission of disease is the passing of an infection to offspring of infected parents. That is to say, the offspring of infected parents may already be infected with the disease at birth such as hepatitis, phthisis, etc. In our life, many infections in nature transmit through both horizontal and vertical modes, such as herpes simplex Chagas' disease, rubella, hepatitis B, and the most notorious, AIDS.

Immunization, either by naturally contracting infection or by the use of a vaccine, is a process that render the body temporarily or permanently resistant to infection. Vaccination is a commonly used method for controlling disease.

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For example, pertussis, measles, influenza, etc. We usually adopt the vaccination to control the spread. Vaccinations have many types, continuous vaccination and impulsive vaccination are two main policies. Continuous vaccination is that people have been vaccinated at birth to protect themselves from disease, the studies can be found in [9,6]. Makinde [9], studied a SIR model for the transmission dynamics of a childhood disease in the presence of a preventive vaccine, and analyzed the vaccination reproductive number for disease control and eradication qualitatively. Impulsive vaccination, only at fixed time sequence we execute effectively the vaccination for the disease, is also an important and effective strategy for the elimination of infectious diseases, and was studied in many literatures. For example, see [13,7,5,1,8,4,3]. The literature on impulsive vaccination dealing with the analysis of disease that are vertically and horizontally transmitted is not extensive, we can see [8,4]. In above mentioned papers, authors almost considered the vaccination of susceptible population. But, in fact, under the situation of disease with vertical transmission, the vaccine treatment should be taken to the newborns who have not been infected by their infectious parents at birth and transfer to the susceptible, and the vaccine treatment also should be considered for the newborns of the susceptible, exposed and the removed under the situation of disease with horizontal transmission. We find that few studies are written on the aspect of the vaccination of newborns.

Incidence rate plays a very important role in the research of epidemic models. Bilinear incidence rate βSI and standard incidence rate $\beta S/N$ (N is the total population size) are frequently used. It is unreasonable to consider the bilinear incidence rate based on the law of mass action, however, as the number of susceptible is large, owing to the number of susceptible with which every infective contact within a certain time is limited. The standard incidence rate may be a good approximation if the number of available partners is large enough and it is not possible to make more contacts than is practically feasible. Combine the two previous approaches by assuming that, if the number of available partners N is low, the number of actual per capita partners is proportional to N, whereas, if the number of available partners is large, there is a saturation effect which makes the number of actual partners constant. These imply that the number of new cases per unit time is saturated with the total population. If the incidence is saturated with the susceptible or the infective, there are two kinds of incidence forms that are used in epidemiological model: $\beta SI/(1+\alpha S)$, $\beta SI/(1+\alpha I)$. Comparing bilinear and standard incidence, saturating incidence may be more suitable for our world.

The aim of this paper is to analyze a class of SEIR epidemic models describing delayed differential equations. In these models, we assume the disease infects vertically and horizontally and has saturation incidence and the fixed latency period ω. We establish the sufficient conditions of extinction and permanence of the disease toward the newborns' continuous and impulsive vaccination, respectively. For continuous vaccination policy, we get the basic reproduction number R_1. If $R_1 \leq 1$, the disease-free equilibrium is globally attractive. Whereas, if $R_1 > 1$, then the disease-free equilibrium is unstable, that is the disease will spread. In impulsive vaccination case we obtain two thresholds R^* and R_* and prove that if $R^* < 1$ then the disease-free periodic solution is globally attractive and if $R_* > 1$ then the disease is permanent. Further, we compare the two policies of vaccination and analyze the two thresholds R^* and R_* by numerical simulation.

2. Model formulation

Cooke and Driessche [2] investigated an SEIRS model with the latent period and the immune period. The consideration of the latent period and the immune period gives rise to a model with the incorporation of delays and integral equation formulations. By neglecting the disease-related rate, the model which was proposed in [2] yields:

$$\begin{cases} \dot{S}(t) = bN(t) - bS(t) - \dfrac{\beta S(t)I(t)}{N(t)} + rI(t-\tau)e^{-b\tau}, \\ E(t) = \displaystyle\int_{t-\omega}^{t} \frac{\beta S(u)I(u)}{N(u)} e^{-b(t-u)}\,\mathrm{d}u, \\ \dot{I}(t) = e^{-b\omega}\dfrac{\beta S(t-\omega)I(t-\omega)}{N(t-\omega)} - (b+r)I(t), \\ R(t) = \displaystyle\int_{t-\tau}^{t} rI(u)e^{-b(t-u)}\,\mathrm{d}u. \end{cases} \tag{2.1}$$