Available online at www.sciencedirect.com

ScienceDirect

Mathematics and Computers in Simulation 79 (2009) 2240–2249

MATHEMATICS AND COMPUTERS IN SIMULATION

www.elsevier.com/locate/matcom

Intraday trade and quote dynamics: A Cox regression analysis

Chad R. Bhatti

NSF VIGRE Postdoctoral Researcher, Department of Mathematics, Tulane University, 6823 St. Charles Ave., New Orleans, LA 70118, United States

Received 17 September 2008; received in revised form 16 December 2008; accepted 16 December 2008
Available online 6 January 2009

Abstract

In this paper we apply the Cox proportional hazards model with an automated forward variable selection algorithm to identify the prominent market microstructure variables affecting the arrival rates of the trade and response quote processes. We use this flexible data-driven modeling approach to empirically examine the informational dynamics of individual securities and the economic similarities in trade and response quote dynamics across samples without imposing a structured relationship on the data.

Keywords: Dependent point processes; Market microstructure; High-frequency finance; TAQ data; Cox proportional hazards model

1. Introduction

The analysis of intraday financial durations has been a topic of widespread interest over the last decade. The availability of high frequency financial data has spurred the development of econometric techniques for the analysis of intraday financial data, and the analysis of the data themselves is a topic of interest to researchers in the area of market microstructure. In this paper we investigate the effects of market microstructure information on the joint process of trade and response quote dynamics—a fundamental economic process in the theoretical market microstructure literature.

Beginning with the Autoregressive Conditional Duration (ACD) model [11], many dynamic duration models have been proposed and applied to analyze intraday financial durations. The primary focus of these dynamic duration models has been to describe the time series dynamics of the intraday financial durations. To this end, many of the dynamic duration models are the duration analogues of popular volatility models, although more recently dynamic proportional hazards models have been proposed [14,16]. The close relationship between dynamic duration models and volatility models is readily apparent in the construction of the ACD model [11] and the Stochastic Conditional Duration (SCD) model [3]. This relationship is also apparent in many of the other dynamic duration models which are variants or generalizations of the ACD model. These variants include generalizations of the distribution used to specify the ACD model such as the generalized gamma ACD model [20] and the Burr ACD model [17], generalizations of the conditional mean specification such as the log ACD model [2,1] and the Box–Cox specification [13], and other time series generalizations such as long memory [18] and regime-switching processes, [25,6,4].

In this paper we deviate from the ACD literature with regard to the usual interest in the time series dynamics of intraday financial durations. Instead, we focus on identifying the market microstructure information defining the

E-mail address: bhatticr@gmail.com.

0378-4754/$36.00
doi:10.1016/j.matcom.2008.12.020

informational dynamics between the trade and response quote processes. This problem is an empirical investigation of the informational market microstructure models common in the theoretical literature [22]. The analysis of this market microstructure problem differs from the ACD literature in two distinct manners. First, the analysis of trade and response quote dynamics considers a pair of intraday financial durations. Second, the response quote duration is subject to right censoring by the trade process. ACD models do not allow for censoring. In this paper we model this pair of financial durations using the Cox proportional hazards (Cox PH) model. The semi-parametric Cox PH model allows for censored durations and provides a degree of flexibility beneficial to empirical modeling which is not provided by ACD models or the competing risks model of Engle and Lunde [10], who also consider the problem of trade and response quote dynamics.

Our work differs from the work of Engle and Lunde [10] in its empirical considerations and the statistical model employed in the investigation of these empirical considerations. Since the Cox PH model is a regression framework, all of the market microstructure information, including the past histories of the processes themselves, must be observable. This model specification facilitates two empirical advantages over the model of [10]. First, it allows the dynamics of the response quote process to be specified in a manner which remains causal in time. The model of [10] requires that the joint likelihood function be factored into a trade density, whose mean dynamics remained causal in time, and a quote density, whose mean dynamics do not remain causal in time. This parametric formulation requires that the mean response quote duration be affected by concurrent trade information, which is contradictory to a theoretical informational model where the specialist observes trading behavior and then reacts to what he has observed. Our approach frees the response quote dynamics of this constraint and hence is more consistent with the theoretical market microstructure literature with regard to the flow of information from the trade process to the quote process.

The second empirical advantage provided by the Cox PH model is the ability to easily and simultaneously consider several hypotheses about market microstructure effects. One can hypothesize several different market microstructure variables which may contain information affecting the trade or response quote processes. However, several of the variables may represent redundant information or be highly correlated. The identification of the best set explanatory variables for each process poses a cumbersome problem. When considering a single explanatory variable, one would compare the different models based on a measure of goodness-of-fit. However, when one wishes to compare models which contain several variables or select an "optimal" set of explanatory variables, the task quickly becomes difficult to manage. In this situation automated variable selection procedures are commonly employed to identify a best subset of explanatory variables. These procedures have not been previously used in the context of intraday duration modeling since the procedures are not easily implemented with latent variable models such as the ACD model, which must be implemented using ad hoc computer programs. The automated variable selection procedure provides both an exploratory and a modeling device. The results from our approach add a statistically sound empirical contribution to the market microstructure literature, in particular for the lesser analyzed response quote duration.

The remainder of this paper proceeds as follows. We begin by defining the trade and response quote durations and the general Cox PH model for these durations in Section 2. In Section 3 we describe the data and define the market microstructure variables affecting intraday trade and response quote dynamics. In Section 4 we describe the automated variable selection procedure, present the estimation results for the individual samples, and investigate common models across samples. We conclude the paper in Section 5.

2. Events, durations, and the general model

Let T_i denote the random trade times and Q_i denote the random quote times. We denote the *first* quote reported after the arrival of a transaction by Q_i^*. We call this quote the *response quote* for transaction T_{i-1}, and note that $Q_i^* > T_{i-1}$ by construction. From the events T_i and Q_i^* we construct two market microstructure durations: the *forward trade duration* defined by $X_i = T_i - T_{i-1}$, and the *forward response quote duration* defined by $Y_i = Q_i^* - T_{i-1}$. The forward response quote duration has a natural market microstructure interpretation as the duration of the specialist's response to the observed trade and any information revealed by the trade. Theoretical models such as [8,9] suggest that the specialist will update is price beliefs based on information inferred from observing individual transactions or sequences of transactions. The more information content in a trade, or a sequence of trades, the more quickly the specialist will change his quote terms in response to the trade.

Not all trades will have an observed forward response quote, frequently the next trade T_i will arrive before the response quote Q_i^*. In this case we say that the forward response quote duration has been censored by the forward