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Block decomposition techniques in the generation of adaptive grids

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Abstract

We consider the problem of the generation of adaptive grids, where, for a given domain Ω, we have to compute a partition that depends on a given map f providing the desired local mesh size. We propose a numerical method for the solution of this problem in the case of planar quadrilateral grids. In particular, a block decomposition method is proposed, by which Ω is first decomposed in several blocks depending on the shape of the domain and on the size map f, and then the whole grid is obtained as the union of the quadrilateral grids on each block. The algorithm concludes with a smoothing step that depends on the size map f. We present some simple numerical examples to test the proposed method.
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Keywords: Adaptive grid; Block decomposition technique; Mapping approach; Grid optimization

1. Introduction

The problem of the construction of a grid on a given domain Ω is usually called grid generation problem, and its solution is given by a suitable partition of Ω. When Ω is a two-dimensional domain this partition is usually made of simple polygons such as triangles and quadrangles, when Ω is a three-dimensional domain this partition is usually made of simple polyhedrons such as tetrahedrals and hexahedrals, see Ref. [6], page 33 for a precise definition.

Grid generation problem arises in several approximation techniques such as finite differences methods and finite element methods, see Refs. [10,21,20] for details. So, the features of the required grid usually depend on the particular problem taken into account. Triangular grids and tetrahedral grids are the most used in the applications based on finite element analyses, and they are also the most studied in the computational geometry field, see Ref. [7] for a complete survey. However, quadrilateral grids and hexahedral grids have some computational advantages with respect to triangular and tetrahedral grids, respectively, see Refs. [1,18] for a detailed discussion. In any case, adaptive grids allow to reduce the computational cost of finite element simulations. In fact, according to the difficulty of the problem taken into account, an adaptive grid is given by elements having different size in different regions of Ω. This information on the size of the elements is usually provided by a user-defined function, i.e. the size map. This map is a real valued function for isotropic grids, and it is a vector-valued function for anisotropic grids, see Ref. [6], page 685 for a detailed discussion.

Quadrilateral and hexahedral grids are more rigid geometric objects than the triangular and tetrahedral grids, respectively. For example, every quadrilateral grid on a two-dimensional domain Ω has always an even number of

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vertices on the boundary $\partial\Omega$ of Ω [15], and every hexahedral grid on a three-dimensional domain Ω has always an even number of faces on $\partial\Omega$ [13]. Thus efficient numerical methods for the generation of quadrilateral and hexahedral grids are not so frequent as those for the generation of triangular and tetrahedral grids, see Ref. [12] for a detailed discussion.

A few general techniques are usually used as the main structure in these methods. The mapping techniques [8,10,4] are very efficient methods, but they can be used only for domains Ω having a simple boundary $\partial\Omega$. The advancing front techniques [2,3] usually produce high quality grids, but they need very involved implementation codes. The spatial decomposition techniques [16,19] compute a high quality grid in the interior of Ω and a lower quality grid near $\partial\Omega$. Finally, the block decomposition techniques [17,15] are able to deal with difficult grid generation problems, but they usually have a high computational cost. All these methods can be specialized for the generation of adaptive grids, see Ref [11] and Ref [6] page 685 for details.

We consider a detailed study of a block decomposition method, which is based on a quite simple idea: the original domain is partitioned in several simple blocks and for each block a different grid is computed, so that the union of the grids on the various blocks gives the required grid on Ω. We note that this decomposition is sometimes given with the description of domain Ω, in fact various solid modeling methods, such as the Constructive Solid Geometry Method and the Cell Decomposition Method, describe a generic complex object Ω combining several simple solid shapes, which are usually called primitives, see Ref. [14], page 346 for details. However, when this decomposition is not provided as a priori information, the computation of the decomposition blocks of Ω is a crucial phase in the overall method. In particular, this method is given by three main phases: (i) the decomposition of domain Ω by a geometric optimization technique, see Ref. [15] for details, (ii) the generation of the grids in the computed blocks by using a mapping approach, i.e. the transfinite interpolation method, see Ref. [8], page 92 for details, (iii) the optimization of the whole grid by a variational grid generation approach, see Ref. [5] for a detailed presentation of this method. In this paper we consider a simple modification of this block decomposition method for the computation of adaptive isotropic quadrilateral grids on domain Ω. In particular, we propose a decomposition of Ω based on the shape of Ω and on the relative extrema of the size map. Thus, starting from this decomposition, the adaptive grid is computed by the procedure explained above, that is the grid generation phase and the optimization phase.

We conclude this section introducing some notations. Let $\mathbf{R}$ be the set of real numbers. Let $x \in \mathbf{R}$, we denote with $\lfloor x \rfloor$ the nearest integer less than or equal to x. Let N be a positive integer, we denote with $\mathbf{R}^N$ the N-dimensional real Euclidean space. Let $\underline{x} = (x_1, x_2, \ldots, x_N)^{\mathrm{T}} \in \mathbf{R}^N$ be a generic vector, where the superscript T denotes the transposition operation. Let $\underline{x}_0, \underline{x}_1, \ldots, \underline{x}_J \in \mathbf{R}^2$, we denote with $\mathcal{S}(\underline{x}_{j-1}, \underline{x}_j)$ the line segment joining $\underline{x}_{j-1}, \underline{x}_j$, $j = 1, 2, \ldots, J$, and with $\mathcal{P}(\underline{x}_0, \underline{x}_1, \ldots, \underline{x}_J)$ the polyline through points $\underline{x}_0, \underline{x}_1, \ldots, \underline{x}_J$. We say that this polyline is closed when $\underline{x}_0 = \underline{x}_J$, and it is simple when no pairs of nonconsecutive segments, of type $\mathcal{S}(\underline{x}_{j-1}, \underline{x}_j)$, $j = 1, 2, \ldots, J$, intersect at any place. Let $A \subset \mathbf{R}^N$ be a subset of $\mathbf{R}^N$ we denote with ∂A the boundary of A, and with $\mathrm{cl}(A)$ the closure of A.

In Section 2 we briefly describe the method proposed in Ref. [5] for the solution of the general grid generation problem. In Section 3 we propose a simple variation of this method for the generation of adaptive quadrilateral grids. In Section 4 we report the results of some numerical experiments. In Section 5 we give some conclusions and some possible future developments of this work.

2. The grid generation method

We describe the block decomposition method presented in Ref. [5], where the following problem is taken into account: given a domain $\Omega \subset \mathbf{R}^2$, a characteristic length h, and some points $\underline{u}_1, \underline{u}_2, \ldots, \underline{u}_\beta \in \partial\Omega$, compute a quadrilateral grid on Ω having edges of length h and $\underline{u}_1, \underline{u}_2, \ldots, \underline{u}_\beta$ as vertices on $\partial\Omega$.

This method is given by three main phases. The first phase computes a block decomposition of domain Ω; more precisely, this domain is approximated by a suitable polygon D, and D is decomposed into subpolygons $D_1, D_2, \ldots, D_N$ by using a geometric optimization technique. The second phase computes a quadrilateral grid on each subpolygon D_n, $n = 1, 2, \ldots, N$; this phase uses a mapping technique and an explicit construction of a quadrilateral grid on the unit square. The third phase computes a smoothed version of the grid obtained from the union of the quadrilateral grids on $D_1, D_2, \ldots, D_N$; this phase is based on some ideas arising from the variational grid generation method. In Section 2.1 we describe the first phase. In Section 2.2 we describe the second phase. In Section 2.3 we describe the third phase.

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