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Reducing a class of polygonal path tracking to straight line tracking via nonlinear strip-wise affine transformation

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Abstract

In this paper a piecewise linear homeomorphism is presented that maps a strictly monotone polygonal chain to a straight line. This mapping enables one to reduce the path tracking task for mobile robots to straight line tracking. Due to the simplicity of the transformation, closed form solutions for the direct and inverse mapping are presented. Furthermore, the transformation also defines a feedback equivalence relation between the original and the transformed system equations of the mobile robot. It is shown that the form of the system equations is preserved and that the transformation essentially maps a car-like robot in the original domain, to a car-like robot in the transformed domain. This enables one to use straight line trackers developed solely for this system, for the tracking of arbitrary strictly monotone polygonal curves. Finally, it is shown that the use of this mapping can also simplify the application of existing path tracking controllers since they only need to track straight line paths. In general, one can eliminate from the existing path controllers all parameters that are needed for non-straight paths, thus obtaining respective simplified controllers. For example, it is shown that a fuzzy path controller with 135 rules can be reduced to an equivalent fuzzy straight line tracking controller with 45 rules.

Keywords: Path tracking; Strip-wise affine transformation; Strictly monotone polygonal chain; Mobile robots

1. Introduction and preliminary concepts

Path tracking is an essential task every autonomous mobile robot has to face in the process of navigation. Precision tracking controllers account for successful steering, obstacle avoidance and overall navigation of a mobile robot. The tracking problem for nonlinear systems, such as mobile robots, can be stated as follows: "Let F be a nonlinear system of the form of Eq. (1),

$$\begin{aligned} \dot{x} &= f(t, x, u), \\ y &= h(t, x, u) \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^n$ is the state vector of the system, $u \in \mathbb{R}^n$ is the input vector and $y \in \mathbb{R}^k$ is the output vector. Let $x_r(t)$ be a feasible reference trajectory in the state space, that is assumed to be given, such that $\dot{x}_r = f(t, x_r, u_r)$ satisfies Eq. (1),

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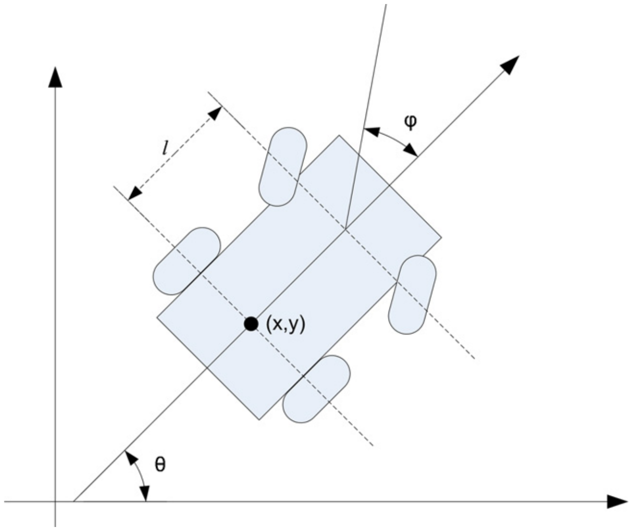

Fig. 1. Definition of the generalized coordinates for the car-like robot.

where u_r is a feasible reference input. Find an appropriate state feedback law $u = u(t, x, x_r, u_r)$ such that x converges asymptotically to x_r." This formulation of the tracking problem is generally known as the *state-feedback state tracking problem* [16] since all state variables are used in the feedback loop. The tracking problem, for mobile robots, can be divided into two further cases; the *path tracking problem*, where a feedback law is needed in order to drive the robot along a reference geometric path, and the *trajectory tracking problem* where the path has a temporal parameterization, i.e. the robot lies on the path at specific times. The most widely used model for the kinematic analysis of wheeled robots is the *car-like robot*:

$$\Sigma : \quad \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \\ \dot{\kappa} \end{bmatrix} = \begin{bmatrix} \cos\theta \\ \sin\theta \\ \kappa \\ 0 \end{bmatrix} u_1 + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u_2 \tag{2}$$

where x, y are the coordinates of the midpoint of the rear axle, θ is the direction of the robot, κ is the steering curvature and u_1, u_2 are the available inputs (see Fig. 1).

The steering curvature is related to the steering angle φ by the formula $\kappa = \tan \varphi / l$, where l is the distance between the front and rear axles. The "standard" model for the car-like robot uses the steering angle as a state (and consequently its derivative as a control) instead of the steering curvature. The former model is simpler and more general in the sense that is does not explicitly take into account the dimensions of the robot. The model is governed by the non-holonomic constraints of *rolling without slipping* and the *rigid body constraint* which take the Pfaffian form:

$$\begin{aligned} &\dot{x}\,\sin\theta - \dot{y}\,\cos\theta = 0 \\ &\dot{x}\,\sin(\theta+\phi) - \dot{y}\,\cos(\theta+\phi) - \dot{\theta} l\,\cos\phi = 0 \end{aligned} \tag{3}$$

Systems of the form of (2) have been extensively studied since they are linear with respect to the inputs $\boldsymbol{u}$. They are known as *control-affine systems* [14]. More specifically, let M be an n-dimensional manifold, $h_0, h_1, \ldots, h_m, m < n$, be vector fields on M, $\boldsymbol{x} \in \mathbb{R}^n$ be the states of the system and $\boldsymbol{u} \in U$ be the inputs. Then a control-affine system is a system of the form:

$$\dot{x} = h_0(x) + \sum_{i=1}^{m} h_i(x) u_i \tag{4}$$

The term $h_0(x)$ is called the *drift* of the system. In the case of the car-like robot the *drift* is identically zero and the system is described by the linear combination of the vector fields h_i on M. Such a system is called *driftless control-affine* or *control-linear system*. The state space of the car-like robot is $X = \mathbb{R}^2 \times S^1 \times [\phi_{\min}, \phi_{\max}]$, and the input space U is