Available online at www.sciencedirect.com

ScienceDirect

Mathematics and Computers in Simulation 79 (2008) 189–200

MATHEMATICS AND COMPUTERS IN SIMULATION

www.elsevier.com/locate/matcom

Application of homotopy analysis method for fin efficiency of convective straight fins with temperature-dependent thermal conductivity

Mustafa Inc

Department of Mathematics, Fırat University, 23119 Elazığ, Turkey

Received 25 March 2007; received in revised form 27 July 2007; accepted 11 November 2007
Available online 28 November 2007

Abstract

In this paper, the homotopy analysis method (HAM) has been used to evaluate the efficiency of straight fins with temperature-dependent thermal conductivity and to determine the temperature distribution within the fin. The fin efficieny of the straight fins with temperature-dependent thermal conductivity has been obtained as a function of thermo-geometric fin parameter. It has been observed that the thermal conductivity parameter has a strong influence over the fin efficiency. The series solution is developed and the reccurance relations is given. Comparison of the results with those of the homotopy perturbation method (HPM) and the Adomian decomposition method (ADM), has led us to significant consequences. The analytic solution of the problem is obtained by using the HAM. The HAM contains the auxiliary parameter $\hbar$, which provides us with a simple way to adjust and control the convergence region of solution series. By suitable choice of the auxiliary parameter $\hbar$, we can obtain reasonable solution for large values of τ and η.

Keywords: Homotopy analysis method; Fin efficiency; Variable thermal conductivity

1. Introduction

Nonlinear phenomena play a crucial role in applied mathematics and physics. Explicit solutions to the nonlinear equations are of fundamental importance. Various methods for obtaining explicit solutions to nonlinear evolution equations have been proposed. The investigation of exact travelling wave solutions to nonlinear evolution equations plays an important role in the study of nonlinear physical phenomena. The wave phenomena observed in fluid dynamics, plasma, elastic media, optical fibers, etc. In the past several decades, both mathematicians and physicists have made significant progression in this direction. Unlike classical techniques, the nonlinear problems are solved easily and elegantly without transforming or linearizing the equation by using HAM [1–3,10,11,18–23]. It provides an efficient explicit solution with high accuracy, minimal calculations and avodiance of physically unrealistic assumptions.

Fins are used to enhance the heat transfer between a solid surface and its convective by Kern and Kraus [15]. Fins are employed to enhance the heat transfer between the primary surface and its convective, radiatingor convective-radiating environment. Aziz and Hug [6], used the regular perturbation method and a numerical solution method to compute a closed form solution for a straight convecting fin with temperature-dependent thermal conductivity. Razelos and Imre [26] considered the variation of the convective heat-transfer coefficient from the base of a convecting fin to its

0378-4754/$32.00 © 2008 IMACS. Published by Elsevier B.V. All rights reserved.
doi:10.1016/j.matcom.2007.11.009

Nomenclature

A_c	cross-sectional area of the fin (m^2)
b	fin length (m)
C	an integral constant
h	heat-transfer coefficient ($W\,m^{-2}\,K^{-1}$)
$\hbar$	auxiliary linear parameter
k	thermal conductivitiy of the fin material ($W\,m^{-1}\,K^{-1}$)
k_a	thermal conductivitiy at the ambient fluid temperature ($W\,m^{-1}\,K^{-1}$)
k_b	thermal conductivitiy at the base tempareture ($W\,m^{-1}\,K^{-1}$)
£	linear operator
$£^{-1}$	inverse operator of £
N	thermo-geometric fin parameter
$\tilde{N}$	nonlinear operator
p	embedding parameter
P	fin parameter (m)
Q	heat-transfer rate (W)
T	tempareture (K)
V	fin volume
x	distance measured from the fin tip (m)

Greek letters

ε	dimensionless parameter describing thermal conductivity
η	fin efficiency
τ	dimensionless coordinate
ϕ	nondimensional stream function
χ_m	two-valued function

tip. A method of temperature correlated profiles is used to obtain the solution of optimum convective fin when the thermal conductivity and heat-transfer coefficient are functions of temperature [31]. Laor and Kalman [16] examined straight, spine and annular fins governed by power law-type temperature dependence of the heat-transfer coefficient. Yu and Chen [33] invastigated the optimal fin length of a convective-radiative straight fin with rectangular profile under convective boundary conditions and variable thermal conductivity. Bouaziz et al. [7] presented the efficiency of longitudinal fins with temperature-dependent thermophysical properties. Chiu and Chen [8] used ADM to evaluate the efficiency and the optimal length of a convective rectangular fin with variable thermal conductivity. Arslantürk [4] and Rajabi [25] used the ADM and HPM to evaluate the efficiency of straight fins with temperature-dependent thermal conductivity and to determine the temperature distribution within the fin, respectively. Lesnic and Heggs [17] applied the ADM to determine the temperature distribution witin a single fin with a temperature-dependent heat-transfer coefficient.

The HAM is developed in 1992 by Liao. Liao [18–23] applied this method to solve many types of nonlinear equations in science and engineering and then, this method has been succesfully applied to solve many other nonlinear evolution equations.The HAM contains an auxiliary parameter $\hbar$ which provides us with a simple way to adjust and control the convergence region of solution series for large values of x and t. Note that the HAM has already been applied for the analytical solution of several other problems [1–3,5,9–14,23,24,27–30,32]. All these problems verify the validity of the HAM. The validity of the HAM is dependent upon whether or not nonlinear problems under consideration contain small parameters. We choose freedom to select related initial approximation by the HAM.

In this paper, the basic idea of HAM is introduced and then it is used to fins with temperature-dependent thermal conducdivity and to determine the temperature distribution within the fin. This problem is solved through the HAM and comparison is made the ADM and the HPM which are obtained by Arslantürk [4] and Rajabi [25], respectively.