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A Bayesian approach to fuzzy hypotheses testing for the estimation of optimal age for vaccination against measles

Neli R.S. Ortega [a], Eduardo Massad [a,b,*], Cláudio José Struchiner [c]

[a] School of Medicine, The University of São Paulo and LIM 01/HCFMUSP, Rua Teodoro Sampaio 115, São Paulo, CEP 05405-000, SP, Brazil
[b] London School of Hygiene and Tropical Medicine, London University, UK
[c] Program of Scientific Computation, Fundação Oswaldo Cruz, FIOCRUZ, Rio de Janeiro, Brazil

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Abstract

Fuzzy Bayesian tests were performed to evaluate whether the mother's seroprevalence and children's seroconversion to measles vaccine could be considered as "high" or "low". The results of the tests were aggregated into a fuzzy rule-based model structure, which would allow an expert to influence the model results. The linguistic model was developed considering four input variables. As the model output, we obtain the recommended age-specific vaccine coverage. The inputs of the fuzzy rules are fuzzy sets and the outputs are constant functions, performing the simplest Takagi–Sugeno–Kang model. This fuzzy approach is compared to a classical one, where the classical Bayes test was performed. Although the fuzzy and classical performances were similar, the fuzzy approach was more detailed and revealed important differences. In addition to taking into account subjective information in the form of fuzzy hypotheses it can be intuitively grasped by the decision maker.

Finally, we show that the Bayesian test of fuzzy hypotheses is an interesting approach from the theoretical point of view, in the sense that it combines two complementary areas of investigation, normally seen as competitive.

Keywords: Bayes test; Fuzzy sets; Measles; Vaccine; Epidemiology

1. Introduction

Measles is one of the most highly infectious and lethal diseases, being responsible for 10% of global mortality from all causes among children aged less than 5 years, which represents approximately 1 million deaths annually [4]. In spite of a very effective vaccine, which averted approximately 1.67 million measles associated deaths in 1996, measles still is the first cause of deaths preventable by vaccines in children.

Previous studies have already demonstrated the necessity of vaccinating between 94% and 98% of the susceptible children to avoid measles outbreaks [20]. However, the optimal age to vaccinate children in a routine immunization calendar is still dependable on a set of variables, characteristic of the target population, like the seroconvertion rate of children below 1 year of age, the presence of maternally derived antibodies, the serostatus of mothers, among others [25].

* Corresponding author at: School of Medicine, The University of São Paulo and LIM 01/HCFMUSP, Rua Teodoro Sampaio 115, São Paulo, CEP 05405-000, SP, Brazil. Tel.: +55 11 3061 7435; fax: +55 11 3061 7382.
E-mail address: edmassad@usp.br (E. Massad).

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Mathematical models have been proposed and applied for the estimation of optimal age to vaccinate children, not only against measles [8,10,25], but also against rubella [12,15]. Those models are of deterministic structure and, despite some attempts to include age-dependence of the force of infection parameter [1], all the available calculations of the optimal age to vaccinate, to the best of our knowledge, have assumed a constant, age-independent force of infection. In addition, the variables which determine the optimal age to vaccinate are usually difficult to determine and usually only fragmentary information on them are available. Therefore, we proposed in the present work, an alternative approach,which combines two powerful techniques to dealing with subjective and/or imprecise information, namely, the Bayesian approach and the fuzzy sets theory.

Several articles can be found in the literature combining the Bayesian approach with ideas from fuzzy sets theory [3,5,6,14,17,21–24]. In this work we apply the method proposed by Taheri and Behboodian [21], who consider the problem of hypotheses testing when the data (observations) are ordinary (crisp) and the hypotheses are fuzzy, such as: θ is approximately one, θ is very high, etc. [21]. We assume that the estimation of optimal age to vaccinate is a decision analysis procedure, which involves aims and constraints (see [13,26] for a discussion on fuzzy decision-making). In this case the aim is to minimize the lifetime expected risk of acquiring measles infection and the constraints include the immunological aspects involving the 'taking' of the vaccine, like the seroconvertion rates, maternally derived antibodies, mother serostatus, among others. Logistic aspects of vaccination will not be considered in this work. We compare our results with those of a previous research of our group on measles vaccination in São Paulo, Brazil [25].

This paper is organized as follows: in Section 2 is discussed the Bayesian approach to fuzzy hypotheses testing, in Section 3 is presented the model performed to estimate the optimal age to vaccinate agaisnt measles, in Section 4 we show our results, whose discussion is presented in Section 5. Finally, we present our conclusions in Section 6.

2. The Bayesian approach to fuzzy hypotheses testing

As mentioned above we will apply the methods proposed by Taheri and Behboodian [21], who consider the problem of hypotheses testing when the data (observations) are ordinary (crisp) and the hypotheses are fuzzy.

As defined by those authors, any hypothesis of the form "H: θ is $H(\theta)$" is said to be a *fuzzy hypothesis*, where $H(\theta)$ is a membership function from the space Θ to [0, 1]. Examples of fuzzy hypothesis include, "θ is approximately 1/2 ", " θ is very low", among others.

The main problem is as follows: Let $X = (X_1, \ldots, X_n)$ be a random sample, with observed value $x = (x_1, \ldots, x_n)$, where X_i has the probability density function (p.d.f.) $f(x_i|\theta)$ with unknown $\theta \in \Theta$, whose prior density is $\pi(\theta)$. Suppose, as in [21], that two membership functions $H_0(\theta)$ and $H_1(\theta)$ are given. We want to test:

$$H_0 : \theta \text{ is } H_0(\theta), \qquad H_1 : \theta \text{ is } H_1(\theta) \tag{1}$$

on the basis of a Bayesian inference.

As we assumed the problem of finding the best age to vaccinate children against measles as a decision analysis problem we must define the space of possible actions (ages of vaccination) A, and the loss function $L(\theta, a) : \Theta \times A \to R$. The loss function specifies the loss when taking action a when the true parameter is θ. Here θ represents some characteristics of the children population like antibodies, seroconvertion, etc.

Let us now assume that θ has prior distribution $\pi(\theta)$ and that $f(x|\theta)$ is the p.d.f of X with fixed $\theta \in \Theta$. Then, the posterior density of θ, $\pi(\theta|x)$, is proportional to its prior and to the p.d.f. of X:

$$\pi(\theta|x) \propto \pi(\theta) f(x|\theta). \tag{2}$$

The Bayesian risk of a wrong decision d (that is, the probability of taking the wrong decision), associated with the prior $\pi(\theta)$, is then [21]:

$$R(\pi, d) = E[R(\theta, d)]. \tag{3}$$

Our aim is, therefore, to minimize the risk by taking the 'optimal' decision d^*, that is

$$R(\pi, d^*) = \inf_{d \in D} R(\pi, d) \tag{4}$$

where D is the space of possible decisions.