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Optimal location of sampling points for river pollution control

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Abstract

Common methods of controlling river pollution include establishing water pollution monitoring stations located along the length of the river. The point where each station is located (*sampling point*) is of crucial importance and, obviously, depends on the reasons for the sample. Collecting data about pollution at selected points along the river is not the only objective; must also be extrapolated to know the characteristics of the pollution in the entire river. In this work we will deal with the optimal location of sampling points. A mathematical formulation for this problem as well as an efficient algorithm to solve it will be given. Finally, in last sections, we will present numerical results obtained by using this algorithm when applied to a realistic situation in a river mouth.

Keywords: Mathematical modelling; Optimal location; Sampling points; River pollution

1. Introduction

For ages, people have used rivers as refuse sites to dispose of the waste which they have generated. As a consequence of the growing industrialization and population explosions in urban areas, wastewater discharges in our rivers have also increased, thus leading to serious water pollution problems. In the last century, developed countries became aware these problems and established strict legislative requirements concerning to the wastewater disposal in rivers. At present, all wastewater discharged into a river must first be treated in a purifying plant, in order to reduce its level pollutants. Depending on its source, wastewater in rivers is classified under two main types: domestic wastewater (that coming from a purifying plant which collect the water from a sewer system) and industrial wastewater (that coming from an industrial plant).

Wastewater purification at each plant must be strong enough so that the river basin is capable of assimilating all the wastewater disposed there. In order to be sure that the river is assimilating the discharges, we have to choose some concrete indicators of pollution levels and design an adequate sampling technique which gives us information about the values of these indicators along the river. For instance, if we want to control pollution in terms of pathogenic microorganisms coming from domestic wastewater, one of the most important indicators is the concentration (units m^{-3}) of faecal coliform bacteria because its concentration in wastewater discharges is much greater than any other microorganism concentrations. A common technique to control the concentration of coliform bacteria in rivers is to divide it into

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several sections, according to the morphology of the river basin and the number, type and location of the discharges, and to take samples of water at one point of each section. The point where the station sampling is located (sampling point) is of crucial importance if we want to obtain representative information about the pollution in the whole section. Thus, the optimal sampling point is that one at which the concentration of coliform bacteria over time is as similar as possible to the mean concentration in the whole section (an overview of the entire sampling process related to water quality monitoring programs can be seen, for example, in the books [7,12], or in the almost exhaustive review [5]).

The main goal of this work is to use mathematical modelling and numerical optimization to obtain the optimal sampling point in each section of a river. There also exist other important design variables for the general problem (such as number of sampling points, sampling frequencies, etc.), but we will only address here the optimal location of characteristic points for water quality sampling. The placement of sampling sites has often been dictated by ease of access, but the best spatial distribution of sampling sites should be determined on the basis of the program goals. The first attempt at producing a method for the optimum placement of sampling points was made in 1971 by Sharp [13] by means of topographical techniques. Two years later, Ward [15] advocated placing sampling stations at critical quality points, but this statement seems not to be optimal (cf. [12] or [8]). In 1977 Lettenmaier and Burges [10] analyzed the problem (including also sampling frequency) with an extended Kalman filter. Their results show that, in general, sampling point placement is far less critical than the number of points used. In more recent times, the problem has been also studied with the help of modern tools such as geostatistical schemes [11], graph theory and simulated annealing methodology [6], or genetic algorithms [9]. What we present here is a more simple and efficient numerical method for finding the optimal sampling locations, combining a partial differential equations system for the water quality model and a derivative-free algorithm for the optimization problem.

So, in next section we will analyze the problem from a mathematical point of view, showing that, under suitable hypotheses, it can be formulated as an optimization problem where the cost function is obtained by solving two one-dimensional hyperbolic boundary value problems. We will deal with the theoretical analysis of the problem in Section 3, and, in Section 4, we will present a numerical algorithm to solve the complete model and to obtain the optimal sampling points. Finally, in last sections, we will present the numerical results obtained in a realistic situation.

2. Mathematical formulation

We take a river L meters in length, and we consider E tributaries flowing into the river and V domestic wastewater discharges coming from purifying plants. Following the diagram in Fig. 1, we suppose the river is divided into N sections, consecutively numbered from the source, and we denote by Δ_i the length of the ith zone ($i = 1, 2, \ldots, N$).

Since we are going to consider only one-dimensional changes along the direction of flow in the river, if we want to control pollution for T seconds, for each $(x, t) \in [0, L] \times [0, T]$, we denote by $\rho(x, t)$ the average coliform concentration in the transversal section, x meters from the source and t seconds from the moment the control is initiated. If we define:

- $a_0 = 0$, $a_{i+1} = a_i + \Delta_i$, for $i = 1, 2, \ldots, N$,
- $c_i(t) = \dfrac{\int_{a_{i-1}}^{a_i} \rho(x,t)\,\mathrm{d}x}{\Delta_i}$, for $i = 1, 2, \ldots, N$.

The problem consists of finding the sampling points $p_i \in [a_{i-1}, a_i]$, for $i = 1, 2, \ldots, N$, such that minimize the cost function:

$$J(p) = \sum_{i=1}^{N} \int_0^T (\rho(p_i, t) - c_i(t))^2\,\mathrm{d}t \quad \text{with } p = (p_1, p_2, \ldots, p_N).$$

If we neglect the effect of molecular diffusion, the coliform concentration is given by solving the following hyperbolic boundary value problem:

$$\left.\begin{array}{ll} \frac{\partial\rho}{\partial t} + u\frac{\partial\rho}{\partial x} + k\rho = \frac{1}{A}\left(\sum_{j=1}^{V} m_j\delta(x - v_j) + \sum_{j=1}^{E} n_j\delta(x - e_j)\right) & \text{in } (0, L) \times (0, T), \\ \rho(0, t) = \rho_0(t) & \text{in } [0, T], \\ \rho(x, 0) = \rho^0(x) & \text{in } [0, L], \end{array}\right\} \quad (1)$$

where