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Valid inequalities for a single constrained 0-1 MIP set intersected with a conflict graph

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A B S T R A C T

In this paper a mixed integer set resulting from the intersection of a single constrained mixed 0–1 set with the vertex packing set is investigated. This set arises as a subproblem of more general mixed integer problems such as inventory routing and facility location problems. Families of strong valid inequalities that take into account the structure of the simple mixed integer set and that of the vertex packing set simultaneously are introduced. In particular, the well-known mixed integer rounding inequality is generalized to the case where incompatibilities between binary variables are present. Exact and heuristic algorithms are designed to solve the separation problems associated to the proposed valid inequalities. Preliminary computational experiments show that these inequalities can be useful to reduce the integrality gaps and to solve integer programming problems.

1. Introduction

It is well-known that the use of strong valid inequalities as cuts can be very effective in solving mixed integer problems. One classical approach to generate these valid inequalities is to study the polyhedral structure of simple sets which occur as relaxations of the feasible sets of those general problems. Two such successful examples are the use of Mixed Integer Rounding (MIR) inequalities, derived from a basic mixed integer set [1,2], and the use of valid inequalities for conflict graphs, resulting from logical relations between binary variables, for solving mixed integer programs [3].

The goal of this paper is to investigate the polyhedral structure of a mixed integer set that results from the intersection of two well-known sets: a simple mixed integer set and the vertex packing set associated with a conflict graph.

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Let X be the set of points $(s,x) \in \mathbb{R} \times \mathbb{Z}^n$ satisfying

$$s + c \sum_{i \in N_1} x_i \geq d, \tag{1}$$

$$x_i + x_j \leq 1, \quad \{i,j\} \in E, \tag{2}$$

$$x_i \in \{0,1\}, \quad i \in N, \tag{3}$$

$$s \geq 0, \tag{4}$$

where $N = \{1,\ldots,n\}$ is the index set of binary variables, and E is the set of pairs of indices of incompatible nodes, $N_1 \subseteq N$, and $c > 0$, $d > 0$. The graph $G = (N,E)$ is known as the conflict graph of pairwise conflicts between binary variables (see [4,3]).

Let $N_0 = N \setminus N_1$. Although the general results and the validity of the inequalities presented in the paper hold for the case where N_0 is empty, some facet-defining conditions need to be adjusted. Therefore, to ease the reading of the paper, N_0 is assumed to be nonempty. When $c > d$, the inequality $s + c\sum_{i \in N_1} x_i \geq d$ can be replaced by the stronger inequality $s + d\sum_{i \in N_1} x_i \geq d$. Thus, henceforward, it is also assumed that $c \leq d$.

Set X is the intersection of two sets: $X = X_{VP} \bigcap X_{SMI}$, where X_{VP} is the vertex packing set defined by (2)–(3), that results by considering the conflict graph $G = (N,E)$, and X_{SMI} is a simple mixed integer set defined by $\{(s,x) \in \mathbb{R} \times \mathbb{B}^{|N_1|}|$ satisfying (1) and (4)$\}$. The convex hulls of X, X_{VP}, and X_{SMI}, are denoted by P, P_{VP}, and P_{SMI}, respectively.

The set X_{SMI} has been intensively used as a relaxation of several mixed integer sets, see [2] for examples. It is well-known that in order to describe P_{SMI}, when $|N_1| \geq \lceil \frac{d}{c} \rceil$, it suffices to add to the defining inequalities (1), (4), $x_i \geq 0$, and $x_i \leq 1, i \in N_1$, the following MIR inequality

$$s + r \sum_{i \in N_1} x_i \geq r \left\lceil \frac{d}{c} \right\rceil, \tag{5}$$

where $r = d - c(\lceil \frac{d}{c} \rceil - 1)$.

On the contrary, a complete description of P_{VP} is not known and since optimizing a linear function over X_{VP} is a NP-hard problem, there is not much hope in finding such a description. Nevertheless, families of valid inequalities are known, see [5–8]. The derivation of inequalities for integer programs based on conflict graphs have also been considered in the past (see [3] for further details).

Although the two sets X_{SMI} and X_{VP} have been intensively considered in the past, to the best of our knowledge, set X has only been considered in a preliminary version of this paper [9]. The most related mixed integer sets considered before are the mixed vertex packing set studied by Atamtürk et al. [10] and the flow set with partial order studied by Atamtürk and Zang [11].

Cuts from valid inequalities for X_{SMI} and X_{VP} are commonly used by researchers using MIP solvers, by identifying these sets as relaxations of the original feasible set. This work aims at deriving new inequalities that can be used when those structures are present simultaneously. Such structures can be found in various mixed integer problems, such as inventory routing, production planning, facility locations, network design, etc. The practical examples that motivated this research stemmed from maritime Inventory Routing Problems (IRPs), see [12,13]. Constraint (1) results from the relaxation of inventory constraints, where s is the stock level at a given location, d is the aggregated demand at that location during a set of periods, c is the vehicle capacity (when several vehicles are considered one may assume this capacity to be constant for all vehicles, otherwise one can take c as the maximum of these capacities) and x_i represents an arc traveled by a vehicle. N_1 is the index set of arcs entering to that particular node. Constraints (2) represent incompatible arcs, that is, arcs that cannot belong to the same route, for instance, due to time constraints. The two sets X_{SMI} (e.g. in [12]) and X_{VP} (e.g. in [13]) were considered as relaxations of the set of feasible solutions previously in such problems. However they have never been considered simultaneously.