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A new exact algorithm for the multi-depot vehicle routing problem under capacity and route length constraints

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ABSTRACT

This article presents an exact algorithm for the multi-depot vehicle routing problem (MDVRP) under capacity and route length constraints. The MDVRP is formulated using a vehicle-flow and a set-partitioning formulation, both of which are exploited at different stages of the algorithm. The lower bound computed with the vehicle-flow formulation is used to eliminate non-promising edges, thus reducing the complexity of the pricing sub-problem used to solve the set-partitioning formulation. Several classes of valid inequalities are added to strengthen both formulations, including a new family of valid inequalities used to forbid cycles of an arbitrary length. To validate our approach, we also consider the capacitated vehicle routing problem (CVRP) as a particular case of the MDVRP, and conduct extensive computational experiments on several instances from the literature to show its effectiveness. The computational results show that the proposed algorithm is competitive against state-of-the-art methods for these two classes of vehicle routing problems, and is able to solve to optimality some previously open instances. Moreover, for the instances that cannot be solved by the proposed algorithm, the final lower bounds prove stronger than those obtained by earlier methods.

1. Introduction

The multi-depot vehicle routing problem (MDVRP) is an important class of vehicle routing problem arising in freight distribution and can be defined as follows. We are given a set of depot locations $\mathcal{D}$ and a set of customer locations $\mathcal{C}$, which are assumed to be disjoint (even if two points share the same physical coordinates, they are still handled as different entities). With every customer $j \in \mathcal{C}$ is associated a demand d_j and a service time s_j. We consider a graph $G = (V, E)$ with $V = \mathcal{D} \cup \mathcal{C}$ and $E = \{\{i, j\} : i, j \in V, i \text{ and } j \text{ not both in } \mathcal{D}\}$. With every edge $e \in E$ is associated a traveling cost c_e. With every depot location $i \in \mathcal{D}$ is associated a fleet of size m_i. The fleet is assumed to be homogeneous with all vehicles having the same capacity Q and having to respect a maximum route length of T units. The route length of a route is the sum of the traveling costs along the edges used by the vehicle plus the service times of each visited customer. The objective is to select a subset of vehicles and to construct routes that respect the capacity and route length constraints, so as to visit each customer exactly once, at minimum traveling cost. The capacitated vehicle routing problem (CVRP) is a particular case of the MDVRP in which $|\mathcal{D}| = 1$, the number of vehicles to be used is exactly m and the maximum route length constraint is relaxed.

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Both problems are $\mathcal{NP}$-hard since they are generalizations of the traveling salesman problem, therefore polynomial-time algorithms are unlikely to exist unless $\mathcal{P} = \mathcal{NP}$ [1]. Despite the computational complexity of these problems, state-of-the-art heuristic methods can find near-optimal solutions in a matter of seconds [2,3].

The literature on exact approaches for the MDVRP is sparse. In fact, most authors have focused on the development of heuristic methods to find good quality solutions quickly [4,2,3]. The most recent exact method reporting results on the MDVRP is that of Baldacci and Mingozzi [5]. The method is based on the additive bounding procedure of Christofides et al. [6] applied to several different relaxations of the problem. Ultimately, the set-partitioning formulation of the MDVRP is solved by means of column generation strengthened with the so-called strong capacity constraints and clique inequalities. They do not consider the inclusion of the route length constraint and so experiments are conducted only on those instances without such requirement. A problem closely related to the MDVRP is the periodic VRP (PVRP), in which the complete planning horizon is subdivided in periods, and vehicle routes cannot be longer than the length of one period. The MDVRP can be formulated as a PVRP by realizing that different depots can be modeled as multiple periods in the context of a PVRP. Therefore, any algorithm that solves the PVRP can also solve the MDVRP. Baldacci et al. [7] proposed an exact algorithm for the PVRP that generalizes their former method for the MDVRP but, as remarked by the authors, does not improve upon their previous results.

With respect to the CVRP, the literature on exact methods is broader. The most efficient exact algorithms for the CVRP are those of Lysgaard et al. [8], Fukasawa et al. [9], Baldacci et al. [10,11]. Lysgaard et al. [8] developed the most efficient branch-and-cut algorithm for the CVRP. They consider a compact two-index vehicle-flow formulation of the problem and use several classes of valid inequalities for which they provide novel and efficient separation algorithms. Their method is able to consistently solve problems with up to 50 customers. Fukasawa et al. [9] developed the first branch-and-cut-and-price algorithm for the CVRP. They consider a set partitioning formulation in which variables represent vehicle routes satisfying the capacity constraint. The routes are allowed to contain cycles, and the pricing sub-problem can be solved efficiently by using dynamic programming. This column generation approach is embedded into a branch-and-cut framework, using several of the valid inequalities introduced in [8]. Their computational results show that their method can scale and solve instances with twice as many customers as the previous exact method of Lysgaard et al. [8]. Baldacci et al. [10,11] introduced a new approach to solve the CVRP based on the additive bounding technique introduced by Christofides et al. [6] on which several different relaxations of the problem are solved sequentially in an additive manner. At last, the set-partitioning formulation of the problem is strengthened with strong capacity cuts and clique inequalities, and solved by column generation. The last part of the algorithms reduces to solve a pure integer linear problem with a reduced number of variables. The methods of Baldacci et al. [10,11] differ mainly in the SPPRC relaxation used. While the former relies on elementary routes, the latter relies now in the so-called *ng*-routes relaxation, a new pricing sub-problem that uses neighborhood structures to forbid cycles visiting nodes that are close to each other. While the method of Baldacci et al. [10] drastically reduced the computing times with respect to the former method of Fukasawa et al. [9], now the method based on the *ng*-routes relaxation proves even faster and is able to solve some instances that the previous method did not.

With respect to algorithmic enhancements for column generation methods for vehicle routing problems, they can be categorized into two main branches. On the one hand, the development of new pricing sub-problems aimed to achieve the so-called elementary bound (the lower bound obtained when the set of feasible routes is restricted to those not containing cycles) without solving the elementary shortest path problem with resource constraints (ESPPRC) at every iteration, which is known to be strongly $\mathcal{NP}$-hard [12]. On the other hand, the development of new families of valid inequalities aimed to strengthen the linear relaxations of the set-partitioning formulations of these problems.

In the first category, authors have focused in the development of relaxations of the original ESPPRC to consider routes with cycles. The shortest path problem under resource constraints and without cycles of length one or two (2-*cyc*-SPPRC), introduced by Houck et al. [13] and later used by Desrochers et al. [14] in the context of the vehicle routing problem with time windows (VRPTW) is an example of such relaxation. In the 2-*cyc*-SPPRC, routes are allowed to contain cycles as long as these cycles do not visit the same node twice with a separation of one intermediate node. The shortest path with resource constraints and without cycles of length k for $k \geq 3$ introduced by Irnich and Villeneuve [15] (k-cyc-SPPRC) is another example, in which routes are now allowed to contain cycles as long as no route visits a customer twice with less than k customers of separation. The decremental state-space relaxation (DSSR) introduced by Righini and Salani [16] achieves the elementary bound by first relaxing the elementarity constraint, and iteratively imposing elementarity on the nodes with cycles, until no more cycles are detected. The *ng*-routes relaxation (*ng*-SPPRC) introduced by Baldacci et al. [11] is a very efficient relaxation that achieves near-elementary routes in very short computing times. The intuition behind the *ng*-routes relaxation is as follows. During a labeling algorithm, cycles are allowed as long as they do not visit a set of forbidden nodes, which is constructed from predefined sets of neighbors of every node, and that intuitively forces cycles to contain nodes that are far from each other. The set of forbidden nodes is updated after every extension of the labels and is made in such a way that the efficiency of the dynamic programming algorithm is not compromised. As a result, very strong lower bounds can be obtained that in many cases coincide with the elementary bounds. Contardo et al. [17] introduced the so-called strong degree constraints (SDC), a new family of valid inequalities that are proved to impose partial elementarity. The addition of a strong degree constraint associated to a certain customer imposes that no variable associated to a route having a cycle on that node will be basic in the LP relaxation. The dual variable associated to such constraint can be used to derive a sharper rule than that of classic elementarity. More recently, Contardo et al. [18] have performed a computational study to assess the