



# Metric inequalities for routings on direct connections with application to line planning<sup>☆</sup>



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## ABSTRACT

We consider multi-commodity flow problems in which capacities are installed on paths. In this setting, it is often important to distinguish between flows on direct connection routes, using single paths, and flows that include path switching. We derive a feasibility condition for path capacities supporting such direct connection flows similar to the well-known feasibility condition for arc capacities in ordinary multi-commodity flows. The condition can be expressed in terms of a class of metric inequalities for routings on direct connections. We illustrate the concept on the example of the line planning problem in public transport and present an application to large-scale real-world problems.

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## 1. Introduction

We consider network design problems in which capacities are installed on paths in order to route a given demand either on direct connections, i.e., on single paths, or on routes that use several paths and hence involve path-switchings. The goal is to minimize a combination of path installation costs, demand routing costs on the edges of the network, and costs for path-switching. We will propose a model that allows to distinguish between direct and path-switching connections in an efficient way.

The direct connection network design problem aims at applications in public and rail transport, where the paths correspond to lines and path-switchings to transfers. We are particularly interested in the line planning problem to design a system of lines that minimizes a weighted sum of line operation costs and a total

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travel time of the passengers, including transfer penalties. Both objectives are already difficult to compute individually, even for a given line plan. In fact, operation costs are only known precisely once a timetable, a vehicle schedule, a duty schedule, and a duty roster have been computed, see Borndörfer, Grötschel, and Jäger [1]. This is impossible to do within a line planning model. Likewise, the travel routes of the passengers depend on many factors, such as the available choices, individual preferences, availability of information, and the behavior of other passengers, see the surveys of Liu, Bunker, and Ferreira [2] and Fu, Liu, and Hess [3]. Transit assignment models that can forecast such routes are on a much more detailed level than line planning methods can currently handle. Of course, it is desirable to close this gap and incorporate ever more sophisticated routing models into line optimization approaches. A major challenge in this area is the development of tractable models to treat transfers, see Schöbel [4] for a comprehensive survey. A first approach to maximize the number of *direct travelers*, i.e., travelers that do zero transfers, was taken by Bussieck, Kreuzer, and Zimmermann [5] (see also the thesis of Bussieck [6]). They proposed an integer programming model on the basis of a “system split” of the demand, i.e., an a priori distribution of the passenger flow on the arcs of the transportation network. The direct travelers approach is therefore a sequential passengers-first lines-second routing method. However, the passenger flow strongly depends on the line plan which is to be computed. Hence, a number of approaches that integrate line planning and passenger routing have been developed. Schöbel and Scholl [7] and Scholl [8] handle travel and transfer times explicitly in terms of a change-and-go graph that is constructed on the basis of all potential lines. Nachtigall and Jerosch [9] propose a model that constructs passenger routes from partial routes of direct connections. These models allow a complete and accurate formulation of travel and transfer time objectives. Their major drawback is the enormous size, which makes these models computationally intractable for large-scale real-world applications.

This paper presents a tractable model to distinguish between direct connection routes and routes that involve path-switchings. Our approach is based on a novel concept of *metric inequalities for direct connections* (*dcmetric inequalities*). Similar to the metric inequalities for the classical multi-commodity flow problem by Iri [10] and Onaga and Kakusho [11], that characterize feasible edge capacities, the dcmetric inequalities completely characterize the feasible path capacities that support a certain direct connection routing. Adding the dcmetric inequalities to a multi-commodity flow demand routing model allows to distinguish between direct connection routes and routes that involve path-switchings without increasing the computational complexity. This approach can be used to solve large-scale real-world line planning problems including a treatment of transfers.

The paper discusses the model at the example of the line planning problem; we therefore speak of direct connections via lines instead of paths. It is structured as follows. After an overview of the basic notation in Section 2, we start in Section 3 with an (explicit) *direct line connection model*, that associates direct connection routes with the corresponding lines. Section 4 derives the *dcmetric inequalities*, that characterize the feasible line capacities. In Section 5, we use these constraints to construct an efficiently solvable *complete direct connection model*. We present an approximative *basic direct connection model* of polynomial size in Section 6 that involves a combinatorial subset of the dcmetric inequalities. This model can be implemented using a branch-and-price approach instead of branch-and-cut-and-price. The pricing of the non-direct connection route variables is discussed in Section 7. Section 9 compares all considered models and relates them to the line planning literature. Section 8 discusses the application of direct connection models to real-world line planning problems. It shows that they work very well in practice. In particular, the basic direct connection model performs similarly to the complete direct connection model.

## 2. Basic notation

We use the following notation. Let  $G = (V, E)$  be an undirected graph representing the infrastructure of a *public transportation network*. The nodes define stations or stops and the edges define streets and tracks,

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