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Time-dependent optimization of a multi-item uncertain supply chain network: A hybrid approximation algorithm☆

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ABSTRACT

We consider the uncertain least cost shipping problem. The input is a multi-item supply chain network with time-evolving uncertain costs and capacities. Exploiting the operational law of uncertainty theory, a mathematical model of the problem is established and the indeterminacy factors are tackled. We use the scaling idea together with transformation approach and uncertainty programming to develop a hybrid algorithm to optimize and obtain the uncertainty distribution of the total shipping cost. We analyze the practical performance of the algorithm and present an illustrative example.

1. Introduction

Although a significant part of the literature consider various static versions of network flow problems in the sense that the network does not change over time, a static network flow problem cannot properly consider the evolution of many practical applications in which there are some time-varying parameters. In many practical applications, such as process systems engineering, information communication technology, and production–distribution planning, the system structure and parameters may be time-varying [1–9]. In such applications, flow values on arcs are not constant but may change over time. Hence, the need for more realistic network models led to the development of multi-period and time-evolving flow problems and they have been applied to a variety of applications.

On the other hand, on many occasions, due to economic reasons or technical difficulties, we have lack of observed data about the unknown state of nature [10]. In actual fact, in reality, some system parameters are not fixed. For instance, the failure of links in communication networks and production–distribution systems will influence links' capacities and costs. Therefore, the capacities and costs of network links may

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not be accurately measured according to probability theory. In this case, some domain experts are invited to evaluate their *belief degree* for events' occurrence. Since human beings usually overweight unlikely events [11], the belief degree may have much larger variance than the real frequency, and this is why we should deal with it by specific tools from uncertainty theory (and not by probability theory or fuzzy theory [12,13,10]). In fact, probability distributions seem inappropriate for describing the uncertainty about indeterminacy factors when sample size is too small or when no samples are available. Independent of probability or fuzzy theories, Liu [12,13] presented uncertainty theory to better describe human decisions in the state of uncertainty and it has provided a new approach to deal with nondeterministic factors in programming problems. We refer to [12] for more details.

Network problems with non-deterministic parameters have been extensively studied because of its substantial applications.

Uncertain network was first introduced by Liu [13]. Gao [14] obtained the uncertainty distribution of the shortest path length using the operational law of uncertainty theory. Gao [15] designed a novel uncertain inference control approach to balance an inverted pendulum system. Gao et al. [16,17] proposed the concept of uncertain graph and investigated graph connectivity and cycle index. Ferland et al. [18] studied a classical multi-commodity flow problem with varying arc capacities. Aneja et al. [19] computed the expected value of multi-commodity flows in random deterministic graphs, where the arcs have probabilistic loss rates. Zhang et al. [20] proposed a method to calculate the Euler index. Lin [21] considered a two-commodity reliability evaluation for a stochastic network problem with node failure. Zhu [22] gave an equation of optimality in uncertain optimal control problem with uncertain process. In addition, Chen et al. [23] extended uncertain integral according to Liu process.

As far as deterministic multi-item problem is concerned, researchers have proposed three basic approaches (resource-directive decomposition, price-directive decomposition and partitioning) for solving the classical min-cost flow problem. Robacker [24] first suggested a resource-directive approach and developed the fundamental concepts of multi-commodity networks. Tomlin [25] developed a decomposition approach. Hosseini [2,6] and Hosseini et al. [4,5] addressed generic multi-item production–distribution problems in time-dependent networks and developed some decomposition-based approaches to handle different variations, but deterministic, of the problem. Jewell [26] developed a partitioning method that maintains dual feasibility and approaches primal feasibility. Vaidya [27] provided interior point algorithm for solving multi-commodity flow problem. Moreover, In order to design efficient algorithms, some researchers used scaling approach to design polynomial time algorithms for the min-cost flow problems. Edmonds et al. [28] introduced a capacity scaling technique to reduce the number of iterations. Rock [29] was the first to propose a cost scaling algorithm for solving the min-cost flow problem. In addition, Schneur et al. [30] and Schneur [31] suggested some efficient scaling techniques for multi-commodity flow problem. Bertsekas [32] proposed ε-relaxation method for the classical min-cost flow problem. Later, Bertsekas et al. [33] introduced auction algorithm to improve the efficiency of the ε-relaxation method.

In classical distribution network approaches, the link costs and capacities are required to have crisp values. However, in practice, capacities may happen not to be fixed. If we have enough data, we may create probability distributions of arc capacities. Unfortunately, sometimes, we cannot obtain probability distributions of arc capacities or arc costs due to influence of indeterminacy factors, such as congestion, failure, maintenance, accidents and weather conditions. As a result, it is improper to employ classical algorithms in these situations. For example, obviously, the length of each path in an uncertain network is uncertain, and we cannot get a shortest path in the normal sense. Instead, what we can investigate is the distribution of the shortest path length [14]. To complement the already existing classical approaches for deterministic network problems, we will make use of techniques from uncertainty and stochastic programming to deal with indeterminacy factors and to derive theoretical models and solution approaches for determining the optimal shipment pattern in uncertain time-evolving multi-item production–distribution systems. To this