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Valid inequalities for the single arc design problem with set-ups

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ABSTRACT

We consider a mixed integer set which generalizes two well-known sets: the single node fixed-charge network set and the single arc design set. Such set arises as a relaxation of feasible sets of general mixed integer problems such as lot-sizing and network design problems.

We derive several families of valid inequalities that, in particular, generalize the arc residual capacity inequalities and the flow cover inequalities. For the constant capacitated case we provide an extended compact formulation and give a partial description of the convex hull in the original space which is exact under a certain condition. By lifting some basic inequalities we provide some insight on the difficulty of obtaining such a full polyhedral description for the constant capacitated case. Preliminary computational results are presented.

1. Introduction

We consider a mixed integer set of the form

$$X = \left\{ (x, z, y) \in \mathbb{R}^n_+ \times \mathbb{B}^n \times \mathbb{Z}_+ \,\middle|\, \sum_{j\in N} x_j \le dy, \quad x_j \le c_j z_j,\ z_j \le y,\ j \in N, y \in \{0, \dots, U\} \right\},$$

where $N = \{1, \dots, n\}$, $\sum_{j\in N} c_j > d$, $0 < c_j < d, j \in N$, d, U and $c_j, j \in N$, are integers, and $U \le \left\lceil \frac{\sum_{j\in N} c_j}{d} \right\rceil$.

The set X is related to two well-known sets: the Single Node Fixed-Charge Network Set (SNFCNS) [1]

$$X_{y=a} = \left\{ (x, z) \in \mathbb{R}^n_+ \times \mathbb{B}^n \,\middle|\, \sum_{j\in N} x_j \le d', \quad x_j \le c_j z_j, \right\},$$

obtained from X by setting y to a constant and the Single Arc Design Set (SADS) [2]

$$X_{z=1} = \left\{ (x, y) \in \mathbb{R}^n_+ \times \mathbb{Z}_+ \,\middle|\, \sum_{j\in N} x_j \le dy, \quad x_j \le c_j, y \in \{0, \dots, U\} \right\},$$

obtained from X by setting $z_j = 1, j \in N$. Therefore the set X can be regarded as an extension of the SNFCNS and the SADS.

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Notice that optimizing an arbitrary objective function over the set $X_{y=a}$, $a \in \{1, \dots, U\}$ is a NP-hard problem (see [1]) which implies that optimizing an objective function over the set X is NP-hard as well.

The set X arises as a relaxation of the feasible set of several mixed integer problems such as lot-sizing and network design problems (see [3]). Next we provide a few examples. In the single-item Lot-sizing with Supplier Selection Problem (LSSP) we are given a set N of suppliers. In each time period one needs to decide lot-sizes and a subset of suppliers to use in order to satisfy the demands while minimizing the costs. For each time period, the set X arises as follows: y represents the integer variable indicating the number of batches to produce, z_j indicates whether the supplier $j \in N$ is selected or not, x_j is the amount supplied by supplier j, d is the size of each batch and c_j is the supplying capacity of supplier j, see [4]. Other examples occur in inventory-routing problems such as the Vendor-Managed Inventory-Routing Problem (see [5]), where, for each time period t, y is an integer variable indicating the number of vehicles used at time t, z_j is a binary variable equal to 1 if the retailer j is served at time t, and 0 otherwise, d is the capacity of each vehicle (assuming a homogeneous fleet), and c_j is the maximum inventory level in retailer j. In [5] the model considers only a single vehicle.

Next we introduce some notations used throughout the paper: for any $S \subseteq N$, $\mu(S) = \lceil \frac{\sum_{j\in S} c_j}{d} \rceil$, and $r(S) = \sum_{j\in S} c_j - (\mu(S)-1)d$. We denote by P, $P_{y=a}$, $P_{z=1}$ the convex hull of X, $X_{y=a}$, $X_{z=1}$, respectively. We use the notation $(a)^+ = \max\{a, 0\}$.

For the SNFCNS, Padberg et al. [1] introduced the flow cover inequalities that can be stated as follows.

Proposition 1.1. *Let S be a cover such that $\sum_{j\in S} c_j = d + \lambda$, $\lambda > 0$ and $\bar{c} = \max_{j\in S} c_j > \lambda$. Then the simple flow cover inequality*

$$\sum_{j\in S} x_j - \sum_{j\in S} (c_j - \lambda)^+ z_j \le d - \sum_{j\in S} (c_j - \lambda)^+, \tag{1}$$

defines a facet of $P_{y=1}$.

It is well known that flow cover inequalities can be lifted. A particular case is the well-known extended flow cover inequality [1]:

$$\sum_{j\in S\cup L} x_j - \sum_{j\in S} (c_j - \lambda)^+ z_j \le d - \sum_{j\in S} (c_j - \lambda)^+ + \sum_{j\in L} (\bar{c}_j - \lambda) z_j,$$

where $\bar{c}_j = \max\{c_j, \bar{c}\}$, $\bar{c} = \max\{c_j | j \in S\}$ and $L \subseteq N \setminus S$. In order to define a facet we need $\bar{c} - \lambda \le c_k \le \bar{c}$ for all $k \in L$.

For the SADS, Magnanti et al. [2] introduce the arc residual capacity inequalities.

Proposition 1.2. *For each $S \subseteq N$ the inequality*

$$\sum_{j\in S} x_j - r(S) y \le (\mu(S) - 1)(d - r(S)),$$

is valid for $X_{z=1}$ and defines a facet of $P_{z=1}$ if S satisfies the following conditions: (i) *if $\mu(S) = 1$, then $|S| = 1$;* (ii) *if $r(S) = d$, then $S = N$.*

They show that the inequalities defining $X_{z=1}$ with the arc residual capacities inequalities suffice to describe $P_{z=1}$.

In a companion paper, Agra and Doostmohammadi [6], discuss the polyhedral structure of the set X when $U = 1$, and its relaxation obtained by removing constraints $z_j \le y, j \in N$. They introduce the set-up flow cover inequalities and provide a full polyhedral description for the constant capacitated case. For the set X with $U = 1$, the set-up flow cover inequalities are obtained from the flow-cover inequalities (1) multiplying the RHS by y:

$$\sum_{j\in S} x_j - \sum_{j\in S} (c_j - \lambda)^+ z_j \le \left(d - \sum_{j\in S} (c_j - \lambda)^+ \right) y. \tag{2}$$

We now describe the contents of this paper. In Section 2 we establish basic properties of P, derive families of facet-defining inequalities which generalize the residual capacity inequalities and flow cover inequalities. In Section 3 we consider the constant capacitated case, provide a compact extended formulation for P, and introduce several valid inequalities in the original space of variables. In addition, we provide the complete characterization of P when the capacities are constant and a particular condition is considered. In Section 4 we discuss the lifting of a class of valid inequalities derived in Section 3. In Section 5 we study the separation problem associated to those valid inequalities derived for the constant capacitated case. Preliminary computational experiments are reported in Section 6.

2. Valid inequalities for P

In this section we investigate the polyhedral structure of P. The following propositions establish basic properties of P and, since they can be easily checked we omit the proofs.

Proposition 2.1. *P is a full-dimensional polyhedron.*