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Splitting procedures for the Mixed Capacitated Arc Routing Problem under Time restrictions with Intermediate Facilities

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ABSTRACT

This paper develops optimal and quick near-optimal splitting procedures for the Mixed Capacitated Arc Routing Problem under Time restrictions with Intermediate Facilities. Splitting procedures are a key component of giant tour-based solution methods for Arc Routing Problems. The optimal and near-optimal splitting procedures are tested within a multi-start constructive heuristic, and a fixed execution-time limit is imposed. Results on benchmark instances show that the constructive-heuristic linked with the new optimal splitting algorithm performs better than the near-optimal versions.

1. Introduction

The Capacitated Arc Routing Problem under Time restrictions with Intermediate Facilities (CARPTIF), first proposed by Ghiani et al. [6], and also referred to as the Arc Routing Problem with Intermediate Facilities under Capacity and Length Restrictions (CLARTPIF), is a variant of the classical Capacitated Arc Routing Problem (CARP) and models residential waste collection. On a mixed network, with one and two-way streets in the case of waste collection, the problem is termed the Mixed CARPTIF (MCARPTIF), first proposed by Willemse and Joubert [14]. The problem considers a graph $\boldsymbol{G} = (\boldsymbol{V}, \boldsymbol{E} \cup \boldsymbol{A})$, where $\boldsymbol{V}$ represents the set of vertices, $\boldsymbol{E}$ represents the set of undirected edges that may be traversed in both directions, and $\boldsymbol{A}$ represents the set of arcs that can only be traversed in one direction. For waste collection, $\boldsymbol{V}$ corresponds to road intersections and dead-ends, while $\boldsymbol{E}$ and $\boldsymbol{A}$ model road segments between vertices. A subset of required edges and arcs, $\boldsymbol{E}_r \subseteq \boldsymbol{E}$ and $\boldsymbol{A}_r \subseteq \boldsymbol{A}$, must be serviced by a fleet of K homogeneous vehicles with limited capacity, Q, that are based at the depot vertex, v_1. The fleet size K can be either fixed, left as a decision variable or treated as unlimited. Vehicles are allowed to unload their waste at any Intermediate Facility (IF) at a cost of λ, and resume their collection routes. At the end of its route a vehicle must first visit an IF before returning to the depot. The set of IFs is modelled in $\boldsymbol{G}$ as $\boldsymbol{\Gamma}$, where $\boldsymbol{\Gamma} \subset \boldsymbol{V}$. The sum of demand on each sub-trip between IF visits may not exceed Q, and unless $v_1 \in \boldsymbol{\Gamma}$, a vehicle has to visit an IF before returning to the depot. Lastly, a route length or time restriction of L is imposed on each vehicle route. For a comprehensive review of the CARP and other Arc Routing Problems we refer the reader to Corberán and Laporte [3] and Corberán and Prins [4].

Since the CARP and all its extensions are $\mathcal{NP}$-hard the most effective methods for solving the problems are based on heuristics and metaheuristics, many of which employ giant tour approaches that rely on tour splitting procedures [10]. Splitting procedures take as input a giant tour and partitions the tour into feasible vehicle routes. In this paper we present optimal and heuristic splitting procedures for the MCARPTIF that can be used in giant tour approaches for the problem. The structure of our optimal splitting procedure provides a substantial improvement in efficiency over the existing CAPRTIF version that we adapted for mixed networks. Fast, near-optimal splitting procedures are also presented. The optimal and near-optimal procedures were tested in a multi-start Route-First–Cluster-Second heuristic on large MCARPTIF instances. Tight time-limits were imposed to reduce the number of starts of the slower, optimal procedures compared to the faster near-optimal procedures. Even with less starts, the Route-First–Cluster-Second heuristic linked with our efficient optimal splitting procedure performed better than the near-optimal splitting versions.

The following is an outline of the remainder of the paper. In the next section we review current splitting procedures for the CARP

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and a few of its extensions. In Section 3 we present the algorithm notation and detailed descriptions of our splitting procedures. In Section 4 we report on computational experiments, focusing on the execution times of the procedures and the difference in partition costs between optimal and near-optimal splitting. We then compare the performance of the different procedures within a multi-start Route-First–Cluster-Second heuristic. The paper concludes in Section 5 with a summary of our main findings and suggestions for future research opportunities.

2. Current splitting procedures for the CARP and CAPRTIF

The first optimal splitting procedure for the CARP was developed by Ulusoy [12] as part of a Route-First–Cluster-Second constructive heuristic. The heuristic is similar to the one of Beasley [1] for the Vehicle Routing Problem. First, edge demands are ignored and a giant tour is constructed servicing all the required edges in $\boldsymbol{G}$. In the second phase, an auxiliary Directed Acyclic Graph (DAG) is constructed whose arcs represent feasible sub-tours of the giant tour, with respect to demand of the sub-tour and vehicle capacity. The DAG is constructed in such a way that the shortest path through the graph gives the optimal partition of the giant tour into feasible vehicle routes. The shortest path can be calculated using any shortest path algorithm. The splitting procedure consists of constructing the DAG, calculating the shortest path through the graph, and decoding the shortest path to retrieve the optimal giant tour partitions. Lacomme et al. [8] and Belenguer et al. [2] develop multi-start Route-First–Cluster-Second heuristics for the CARP and Mixed CARP (MCARP), respectively, whereby different giant tours are constructed and partitioned, and the best returned as the final solution.

Ghiani et al. [7] develop a splitting procedure, similar to CARP versions, for the Capacitated Arc Routing Problem with Intermediate Facilities (CARPIF). Their procedure calculates the optimal placement of Intermediate Facility (IF) visits within a route. The problem allows inter-route offloads so that collected demand between IF visits never exceeds vehicle capacity, but it does not impose route duration limits. As such, a solution always consists of only one route. When a route duration limit is imposed the problem generalises to the CAPRTIF. To solve the problem Ghiani et al. [6] develop a splitting procedure that constructs two DAGs. The first consists of multiple source and destination vertices, each representing a start- and end-edge of a sub-tour in the giant tour. Shortest paths through the DAG between the sources and destinations represent the optimal placement of IFs in all possible sub-tours. The shortest path costs, calculated using a shortest path algorithm, are then used to construct a second DAG whose shortest path represents the optimal partition of the giant tour into vehicle routes. The optimal placement of IFs in each route is traced back to the shortest paths in the first DAG. The splitting procedure of Ghiani et al. [6] can be applied as-is to giant tours on mixed networks. A solution for the MCARPTIF can thus be obtained by combining the Route-First phase of Belenguer et al. [2] for the MCARP to construct a giant tour on a mixed network, and then applying the CARPTIF splitting procedure of Ghiani et al. [6] for the Cluster-Second phase.

To improve the efficiency of splitting procedures Lacomme et al. [8] develop a compact procedure for the CARP that does not explicitly construct the DAG. Instead, the shortest path through the DAG is directly calculated when scanning sub-tours for their feasibility with respect to vehicle capacity limits. Their version also accounts for a secondary objective of minimising fleet size. The compact version is exclusively used in Memetic Algorithms for the CARP [8,9,11] and MCARP [2], which are currently some of the most effective solution methods for the problems. Memetic Algorithms are metaheuristics based on genetic algorithms enhanced with local search procedures. Chromosomes are encoded as giant tours and an optimal splitting procedure is used to determine chromosome fitness each time a new chromosome is evaluated. An efficient splitting procedure is critical for the applications since chromosome evaluation occurs tens of thousands of times during the MA's execution.

In this paper we extended the compact splitting version of Lacomme et al. [8] to the MCARPIF. We then further extended this version to deal with the MCARPTIF and show that it provides a substantial improvement in efficiency over the version of Ghiani et al. [6]. We also developed two quick heuristic splitting procedures, one that greedily inserts IF visits into sub-tours and then calculates the route partitions and a second that employs a next-fit bin-packing procedure.

3. New splitting procedures

Before presenting our splitting procedures we first describe the graph transformation of $\boldsymbol{G}$ and introduce the basic algorithm notation. Consistent with Belenguer et al. [2] and Lacomme et al. [8], the graph $\boldsymbol{G}$ is transformed into a fully directed graph, $\boldsymbol{G}^* = (\boldsymbol{V}, \boldsymbol{A}^*)$. CARPTIF splitting procedures can then be used as-is on the MCARPTIF, and the other way around. Required arcs, $\boldsymbol{A}_r$, and edges, $\boldsymbol{E}_r$, of $\boldsymbol{G}$ correspond in $\boldsymbol{G}^*$ to a subset $\boldsymbol{R} \subseteq \boldsymbol{A}^*$ of required arcs. Each arc, $u \in \boldsymbol{R}$, has a demand, $q(u)$, a collection cost, $w(u)$, and a pointer, $inv(u)$, to the arc between the same vertices but in the opposite direction. Each required arc in the original graph, $\boldsymbol{G}$, is coded in $\boldsymbol{R}$ by one arc, u, with $inv(u) = 0$, while each required edge is encoded as two opposite arcs, u and v, such that $inv(u) = v$ and $inv(v) = u$. The depot is modelled by including in $\boldsymbol{A}^*$ a fictitious loop, σ, with zero deadheading and service cost. Similarly, the set of IFs are modelled in $\boldsymbol{A}^*$ as a set of dummy arcs, $\boldsymbol{I}$, such that each IF in $\boldsymbol{\Gamma}$ is modelled as a fictitious loop, $\Phi_i \in \boldsymbol{I}$, and Φ_i also have zero deadheading and service cost. The cost of the shortest path from arc u to arc v, which excludes the costs of deadheading u and v, is given by $D(u, v)$, which is pre-calculated for all arcs in $\boldsymbol{A}^*$. Shortest paths can be efficiently calculated using a modified version of Dijkstra's algorithm, and may also incorporate forbidden turns and turn-penalties [8]. The best IF to visit after servicing arc u and before servicing arc v can be pre-calculated using

$$\Phi^*(u, v) = \arg\min\{D(u, k) + D(k, v) : k \in \boldsymbol{I}\}, \tag{1}$$

$$\mu^*(u, v) = D\big(u, \Phi^*(u, v)\big) + D\big(\Phi^*(u, v), v\big) + \lambda, \tag{2}$$

where $\Phi^*(u, v)$ gives the best IF to visit, and $\mu^*(u, v)$ gives the cost of the visit, including the unloading cost, λ, and deadheading costs. We denote by $\boldsymbol{S}$ the giant tour to be partitioned, which consists of a list of tasks, $[S_1, \ldots, S_{|\boldsymbol{S}|}]$. It is assumed that the shortest path is always followed between consecutive tasks and only arcs in $\boldsymbol{R}$ are included in $\boldsymbol{S}$, thus it contains no depot or IF dummy arcs as these are implicitly accounted for by the splitting procedures. Sub-tours in $\boldsymbol{S}$ are denoted as $\boldsymbol{S}_{i \to j} = [S_i, \ldots, S_j]$ where $1 \le i < j \le n$ and $n = |\boldsymbol{S}|$. A single MCARPIF or MCAPRTIF route is a list of tasks that always starts with the dummy depot task, ends with a dummy IF and depot task, and may include inter-route IF visits. The list of tasks between dummy arcs represent subtrips and the load collected on a subtrip may not exceed Q. For the MCAPRTIF, the total cost of a route, including all task service costs, deadheading costs between tasks and IF visit costs, may not exceed L.

3.1. Splitting procedures for the MCARPIF

The first splitting procedure that we present is an MCARPIF adaption of the CARP procedure developed by Lacomme et al. [8]. Recall that splitting procedures use $\boldsymbol{S}$ to construct an auxiliary DAG, $\boldsymbol{H}$, in such a way that its shortest path represents the optimal