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A Hotelling queue model with probabilistic service

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ABSTRACT

We study the pricing problem for a firm with two servers where heterogeneous customers can choose between deterministic service and probabilistic service. We find that different queueing priority policies do not affect the firm's revenue but affect the firm's optimal pricing strategies. Specifically, when the flexible customers (who choose probabilistic service) have a high priority, the optimal price of the deterministic service could be lower than the one of the probabilistic service in a small or moderate market.

1. Introduction

Probabilistic selling has attracted increasing attention from consumers, the media, and academia. Under this strategy, "the seller creates probabilistic goods using the seller's existing distinct products or service and offers such probabilistic goods/service to potential buyers as additional purchase choices" [8]. For example, a restaurant selling two different dinner sets, named set A and set B, may offer an additional probabilistic set with a lower price, which can be either set A or B, but a customer who orders this probabilistic set does not know which set will be delivered.

The probabilistic selling strategy is also involved in service systems. The classic example is the "Priceline" model, which has been discussed in many articles and books [24,8,9]. The researchers focus on the various aspects of the business model of Priceline, such as online bidding, channel coordination, and the impact of the probabilistic selling on traditional channels. We explore the strategy of the probabilistic selling model under different service priority policies, to find which policy is more suitable for the service provider. Our study can be referred for many service industries, such as restaurant, theater, advertising, hair-dressing and others.

This paper is mainly linked to recent modeling work on probabilistic goods/service. Marketing scholars have also shown interest in probabilistic selling. Jiang [19] studies the opaque products in a Hotelling model [16]. The author assumes that the flight information is hidden for the opaque products, but consumers expect an equal probability to obtain each product. Fay and Xie [8] introduce a patented system and methods on how to create probabilistic products and facilitate probabilistic selling in practice. Similar to Jiang [19], the authors assume that a monopolist offers two products distributed on a Hotelling line, and compare traditional selling with probabilistic selling. They find that the probabilistic selling strictly increases the firm's revenue if the production costs are sufficiently low. Shapiro and Shi [25] study the opaque selling in competitive settings and show that opaque selling enables sellers to discriminate the prices between different consumers who are or who are not sensitive to product/service characteristics. Such discrimination can also benefit the sellers and thus intensifies the competition. Fay [7] constructs an opaque selling model in which channel considerations are investigated by considering various of contracts between service providers and an opaque intermediary. The author finds that with sufficient brand loyalty, an opaque good can help increase firm's revenue. Many other studies also focus opaque or probabilistic selling in monopoly and competition case [12,26,18,5]. Recently, Rice et al. [23] compare probabilistic selling with markdown selling

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and show that probabilistic selling can increase revenue from full-price sales and reduce the magnitude of discounts, the stockouts and the amount of excess inventory. Huang and Yu [17] study the opaque selling for boundedly rational customers and find that opaque selling may soften the price competition and increase the industry profits as a result of consumer bounded rationality.

Our study is also closely related to the literature studying customer strategic behavior in priority queues. Naor [21] is a pioneer of studying customer equilibrium queueing strategy and how to regulate arrivals through pricing in an observable $M/M/1$ queue. Many works appear after that, studying customers queueing strategy in different priority systems and under different delay information. Among them, Balachandran [3] studies the equilibrium behavior in priority queues; Adiri and Yechiali [1] study a preemptive queueing model with two priority classes; Fayolle et al. [10] study a discriminatory processor sharing (DPS) model; and Koenigsberg [20] examines the preemptive non-resume priority model with heterogeneous customer classes. Alperstein [2] extends the model of Adiri and Yechiali [1] to several priority levels managed by a profit maximizing server. Alperstein [2] finds that the profit increases with the number of priority levels and that for an unbounded number of such levels, customers' surplus is 0. Hassin and Haviv [14] extend the set of possible Nash equilibrium strategies to include the mixed strategies of the threshold type. Other works studying customer strategic behavior in priority queues include Beja and Sid [4], Ghanem [11], Dolan [6], Whang [27], Radhakrishnan and Balachandran [22]. More works in this stream can be found in two excellent survey books [15,13].

Our work contributes to the literature by studying a two-server queueing system with probabilistic service involved. Customers have different tastes over two servers' service and such heterogeneity is represented with a Hotelling line model. To maximize the utility, a customer can choose among three options: to join one of the servers, to choose a probabilistic service, or to balk from the system. A deterministic customer (who chooses deterministic service) will join the server that the customer chooses. However, a flexible customer (who chooses probabilistic service) will be placed to one of the servers randomly with equal chances. We study the optimal pricing strategy of the deterministic service and the probabilistic service for the firm. We consider three priority policies: (1) first come first serve (FCFS); (2) the deterministic customers have a high priority (DCHP); (3) the flexible customers have a high priority (FCHP). We are particularly interested in the impact of the priority policy on the firm's pricing strategy.

2. Hotelling queue model

We consider a Hotelling queue model with two servers owned by one firm. A linear city is assumed in the interval [0, 1]. Consumers are symmetrically distributed along this interval. Each server are located at each extreme point which provides the same kind of service.

Under each of the three queueing priority policies, we study a Stackelberg game between the firm and customers, in which the firm determines the deterministic service price and the probabilistic service price, and each customer reacts by making a decision: join one of the two servers, choose the probabilistic service or balk from queue. The firm maximizes its revenue while each customer maximizes his/her utility which is equal to the service reward taking away the service price and the sojourn cost.

2.1. Customer reward

In a Hotelling model, the customer reward is assumed to decrease with the distance between the customer and the server. The customer reward of the deterministic service from a server, can be set as a linear function of the distance:

$$R = 1 - tx, \tag{1}$$

where t is the unit reward loss, and x is the distance between the location of the customer and the location of the server.

A customer who chooses the probabilistic service is randomly allocated to one of the two servers, such that the expected distance is equal to $\frac{1}{2}$ for a customer who is in any location of the Hotelling line. As a result, the reward of the probabilistic service is equal to $R = 1 - t/2$, where $0 \le t \le 2$ because the reward is supposed to be larger than or equal to 0.

2.2. Customer utility in Hotelling queue model

We assume that customers arrive according to a Poisson process with arrival rate Λ. The service time of each server is exponentially distributed with service rate μ. As a result, each server with customer arrival is an $M/M/1$ queue system. A customer utility U equals to the service reward R, taking away the price that customer needs to pay (could be the price p_d of the deterministic service, or the price p_p of the probabilistic service) and the expected sojourn cost, which is assumed to be a linear function of the expected sojourn time (could be the expected sojourn time W_d of the deterministic service, or the expected sojourn time W_p of the probabilistic service). The sojourn cost per unit time is denoted by θ. We assume that the queues are invisible, i.e., customers do not know the queue lengths before joining the queues, and customer will not quit once they join one of the queues.

Denote the server at location 0 by server 1, and the server at location 1 by server 2. Customers are symmetrically distributed along this interval. Denote $F(x)$ as the cumulative distribution function (CDF) of the customer's location and $f(x)$ as the probability density function (PDF).

Let U_d^i be the utility that a customer located at x chooses a deterministic service from server i, $i = 1, 2$,

$$U_d^1 = 1 - tx - p_d - \theta W_d, \tag{2}$$

$$U_d^2 = 1 - t(1 - x) - p_d - \theta W_d. \tag{3}$$

Let U_p be the utility that a customer chooses a probabilistic service,

$$U_p = 1 - t/2 - p_p - \theta W_p. \tag{4}$$

As we assume that customers are symmetrically distributed along the interval (0, 1), this model is actually equivalent to a model in which there is only one server, and the customers are distributed along in the interval (0, 1/2). Therefore, we can tag server 1 to discuss the customer utility etc.

Given a price, customers will join service if only their utilities are greater or equal to 0, such that $U_d^1 \ge 0$. Given p_d, denote $\underline{x}$ by the solution of $U_d^1 = 0$. Since $F(\cdot)$ is the cumulative distribution function of customers' location and customers are symmetrically distributed between the two symmetric servers (one is at location 0 and the other at 1). Then $F(\underline{x})$ can also be considered as the market share of customers who join the deterministic service of server i, $i = 1, 2$. In this way, $F(\underline{x})\Lambda$ is the actual arrival for the deterministic service of each server.

As the two servers are symmetric and the firm makes the centralized decisions and we denote by z the market share of total actual arrivals, $z/2$ is the market share of arrivals of each server (including deterministic customers and flexible customers) satisfying $U_p = 0$ and $(z/2 - F(\underline{x}))\Lambda$ is the actual arrival for