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On strong KKT type sufficient optimality conditions for nonsmooth multiobjective semi-infinite mathematical programming problems with equilibrium constraints

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ABSTRACT

In this paper, we consider a nonsmooth multiobjective semi-infinite mathematical programming problems with equilibrium constraints (MOSIMPECs). We introduce the concept of Mordukhovich stationary point for the nonsmooth multiobjective semi-infinite mathematical programming problems with equilibrium constraints in terms of the Clarke subdifferentials. Further, we establish that the M-stationary conditions introduced in this paper are strong KKT type sufficient optimality conditions for the nonsmooth multiobjective semi-infinite mathematical programming problems with equilibrium constraints under generalized invexity assumptions. We also illustrate our result with an example.

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1. Introduction

Kanzi [18] derived strong Karush–Kuhn–Tucker necessary and sufficient optimality conditions for a nondifferentiable multiobjective semi-infinite programming problems with Lipschitzian data under invexity assumptions. Recently, Mishra and Jaiswal [22] defined a semi-infinite mathematical programming problem with equilibrium constraints (SIMPEC) and established optimality conditions and duality for the SIMPEC.

A semi-infinite programming problem (SIP) is an optimization problem on a feasible set described by infinitely many of inequality constraints. We can find many applications of SIP in different fields such as Chebyshev approximation, robotics, mathematical physics, engineering design, optimal control, transportation problems, fuzzy sets, cooperative games, robust optimization, etc. (see, Hettich and Kortanek [17] and Polak [29]). We refer to [13,19,31–34] for more details and applications related to SIP. A semi-infinite multiobjective optimization problem is the simultaneously minimization of finitely many scalar objective functions subject to an arbitrary (possibly infinite) set of constraint functions. We refer to the recent results [7,10,11] and the references therein for more details related to semi-infinite multiobjective optimization problem.

Mathematical programs with equilibrium constraints (MPECs) have drawn attention of researchers in the recent years [2,8,9,14–16,20,27,23,35,36]. Mathematical programs with equilibrium constraints arise frequently in various real world problems, e.g., in chemical process engineering [30] and hydro-economic river basin model [6].

In this paper, we consider the following multiobjective semi-infinite mathematical programming problem with equilibrium constraints (MOSIMPEC):

$$
\begin{aligned}
\text{(MOSIMPEC)}\quad & \min\,(f_1(x),\ldots,f_m(x))\\
& \text{subject to: } g_j(x)\le 0,\quad j\in J,\qquad h(x)=0,\\
& G(x)\ge 0,\ H(x)\ge 0,\quad \langle G(x),H(x)\rangle = 0,
\end{aligned}
$$

where J is an arbitrary index set, $f_i:\mathbb{R}^n\to\mathbb{R}$ and $g_j:\mathbb{R}^n\to\mathbb{R}\cup\{+\infty\}$ are locally Lipschitz functions. Also we assume that $h:\mathbb{R}^n\to\mathbb{R}^p$, $G:\mathbb{R}^n\to\mathbb{R}^l$ and $H:\mathbb{R}^n\to\mathbb{R}^l$ are functions with locally Lipschitz components.

To the best of our knowledge, there are only a few paper on optimality conditions for multiobjective mathematical programming problem with equilibrium constraints (MOMPEC); see, Bao et al. [2] and Mordukhovich [26].

Motivated by the works of Kanzi [18], Movahedian and Nobakhtian [27] and Mishra and Jaiswal [22], we extend the

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definition of Mordukhovich stationary point (M-stationary point) to the MOSIMPEC in terms of the Clarke subdifferential. Further, we derive strong KKT type sufficient optimality conditions for the MOSIMPEC under pseudoinvexity and quasiinvexity assumptions.

The paper is organized as follows: in Section 2, we give some preliminaries, definitions and results. In Section 3, we establish strong KKT type sufficient optimality conditions for the MOSIMPEC under pseudoinvexity and quasiinvexity assumptions.

2. Definitions and preliminaries

In this section, we give some preliminary definitions and results, which will be used in the sequel.

The following concepts and results are taken from Clarke [12]:

Let $x \in \mathbb{R}^n$ and let $f : \mathbb{R}^n \to \mathbb{R}$ be a locally Lipschitz function. The Clarke directional derivative of f at x in the direction $v \in \mathbb{R}^n$, and the Clarke subdifferential of f at x are respectively given by

$$f^o(x; v) = \limsup_{y \to x, t \downarrow 0} \frac{f(y + tv) - f(y)}{t},$$

$$\partial_c f(x) = \left\{ \xi \in \mathbb{R}^n : f^o(x; v) \geq \langle \xi, v \rangle, \ \forall v \in \mathbb{R}^n \right\}.$$

Theorem 1. *Let f and g be functions from $\mathbb{R}^n$ to $\mathbb{R}$ which are Lipschitz near $\bar{x}$. Then,*

$$f^o(\bar{x}; v) = \max\{\langle \xi, v \rangle : \xi \in \partial_c f(\bar{x})\},$$

$$\partial_c(\lambda f + g)(\bar{x}) \subset \lambda \partial_c f(\bar{x}) + \partial_c g(\bar{x}), \quad \forall \lambda \in \mathbb{R}.$$

Theorem 2. *Let f be function from $\mathbb{R}^n$ to $\mathbb{R}$ which are Lipschitz near $\bar{x}$. Then, the function $v \to f^o(\bar{x}; v)$ is finite, positively homogeneous, subadditive on $\mathbb{R}^n$ and $\partial_c f(\bar{x})$ is a nonempty, convex, compact subset of $\mathbb{R}^n$.*

The following definitions are taken from Mishra and Giorgi [21].

Definition 1. Let $\eta : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$ be a kernel function and let $f : \mathbb{R}^n \to \mathbb{R}$ be locally Lipschitz on $S \subset \mathbb{R}^n$. Then, f is said to be:

(i) quasiinvex at $\bar{x}$ with respect to η if for any $x \in S$ and any $\xi \in \partial_c f(\bar{x})$, one has

$$f(x) \leq f(\bar{x}) \Longrightarrow \langle \xi, \eta(x, \bar{x}) \rangle \leq 0,$$

(ii) pseudoinvex at $\bar{x}$ with respect to η if for any $x \in S$ and any $\xi \in \partial_c f(\bar{x})$, one has

$$\langle \xi, \eta(x, \bar{x}) \rangle \geq 0 \Longrightarrow f(x) \geq f(\bar{x})$$

(iii) strictly pseudoinvex at $\bar{x}$ with respect to η if for any $x \in S$, $x \neq \bar{x}$ and any $\xi \in \partial_c f(\bar{x})$, one has

$$\langle \xi, \eta(x, \bar{x}) \rangle \geq 0 \Longrightarrow f(x) > f(\bar{x}).$$

The following concept of efficiency was introduced in [28]. For recent developments in the field of vector optimization, we refer to the monograph [1] and the references therein.

Definition 2. Let S be a feasible region of MOSIMPEC. A vector $\bar{x} \in S$ is said to be efficient solution of the MOSIMPEC if for all $x \in S$, one has

$$f(x) - f(\bar{x}) := (f_1(x) - f_1(\bar{x}), \ldots, f_m(x) - f_m(\bar{x})) \notin -\mathbb{R}^m_+ \setminus \{0\}.$$

Definition 3. Let S be a feasible region of MOSIMPEC. A vector $\bar{x} \in S$ is said to be weak efficient solution of the MOSIMPEC if for all $x \in S$, one has

$$f(x) - f(\bar{x}) := (f_1(x) - f_1(\bar{x}), \ldots, f_m(x) - f_m(\bar{x})) \notin -int\mathbb{R}^m_+ \setminus \{0\}.$$

Given a feasible vector $\bar{x}$ for the MOSIMPEC, we define the following index sets:

$$J(\bar{x}) := \{j \in J : g_j(\bar{x}) = 0\},$$
$$\alpha := \alpha(\bar{x}) = \{i = 1, 2, \ldots, l : G_i(\bar{x}) = 0, H_i(\bar{x}) > 0\},$$
$$\beta := \beta(\bar{x}) = \{i = 1, 2, \ldots, l : G_i(\bar{x}) = 0, H_i(\bar{x}) = 0\},$$
$$\gamma := \gamma(\bar{x}) = \{i = 1, 2, \ldots, l : G_i(\bar{x}) > 0, H_i(\bar{x}) = 0\},$$
$$T^+ := \{i : \lambda_i^h > 0\}, \quad T^- := \{i : \lambda_i^h < 0\},$$
$$\beta^+ := \{i \in \beta : \lambda_i^G > 0, \lambda_i^H > 0\},$$
$$\beta_G^+ := \{i \in \beta : \lambda_i^G = 0, \lambda_i^H > 0\},$$
$$\beta_G^- := \{i \in \beta : \lambda_i^G = 0, \lambda_i^H < 0\},$$
$$\beta_H^+ := \{i \in \beta : \lambda_i^H = 0, \lambda_i^G > 0\},$$
$$\beta_H^- := \{i \in \beta : \lambda_i^H = 0, \lambda_i^G < 0\},$$
$$\alpha^+ := \{i \in \alpha : \lambda_i^G > 0\}, \quad \alpha^- := \{i \in \alpha : \lambda_i^G < 0\},$$
$$\gamma^+ := \{i \in \gamma : \lambda_i^H > 0\}, \quad \gamma^- := \{i \in \gamma : \lambda_i^H < 0\}.$$

3. Strong KKT sufficient optimality conditions

The following definition is an extension of Definition 3.1 of Movahedian and Nobakhtian [27] for the MOSIMPEC in terms of the Clarke subdifferential.

Definition 4 (*MOSIMPEC M-Stationary Point*)**.** A feasible point $\bar{x}$ of MOSIMPEC is called MOSIMPEC Mordukhovich stationary point (MOSIMPEC M-stationary point) if there exist $\lambda = (\lambda^h, \lambda^G, \lambda^H) \in \mathbb{R}^{p+2l}$, $\theta_i > 0$, $i \in \{1, \ldots, m\}$ and $\lambda_j^g \geq 0, j \in J(\bar{x})$, with $\lambda_j^g \neq 0$ for at most finitely many indexes such that the following conditions hold:

$$0 \in \sum_{i=1}^{m} \theta_i \partial_c f_i(\bar{x}) + \sum_{j \in J(\bar{x})} \lambda_j^g \partial_c g_j(\bar{x}) + \sum_{i=1}^{p} \lambda_i^h \partial_c h_i(\bar{x})$$
$$- \sum_{i=1}^{l} [\lambda_i^G \partial_c G_i(\bar{x}) + \lambda_i^H \partial_c H_i(\bar{x})],$$
$$\lambda_\gamma^G = 0, \quad \lambda_\alpha^H = 0, \quad \text{either } \lambda_i^G > 0, \ \lambda_i^H > 0 \text{ or}$$
$$\lambda_i^G \lambda_i^H = 0, \ \forall i \in \beta.$$

In the following theorem we prove that the MOSIMPEC M-stationary conditions turn into a strong KKT type sufficient optimality conditions for weakly efficient solution of the MOSIMPEC.

Theorem 3. *Let $\bar{x}$ be a MOSIMPEC M-stationary point. Suppose that each f_i $(i = 1, \ldots, m)$ is pseudoinvex at $\bar{x}$, $g_j (j \in J(\bar{x}))$, $h_i (i \in T^+)$, $-h_i$ $(i \in T^-)$, $G_i (i \in \alpha^- \cup \beta_H^-)$, $-G_i$ $(i \in \alpha^+ \cup \beta_H^+ \cup \beta^+)$, $H_i (i \in \gamma^- \cup \beta_G^-)$, $-H_i (i \in \gamma^+ \cup \beta_G^+ \cup \beta^+)$ are quasiinvex at $\bar{x}$ with respect to a common kernel η. If $\alpha^- \cup \gamma^- \cup \beta_G^- \cup \beta_H^- = \phi$, then $\bar{x}$ is a weakly efficient solution for MOSIMPEC.*

Proof. Suppose on the contrary that $\bar{x}$ is not a weakly efficient solution for MOSIMPEC. Then there exists a feasible point x for MOSIMPEC such that

$$f_i(x) < f_i(\bar{x}) \quad \forall i = 1, \ldots, m.$$

Since each f_i is pseudoinvex with kernel η, we have

$$\langle \xi_i, \eta(x, \bar{x}) \rangle < 0, \quad \forall \xi \in \partial_c f_i(\bar{x}). \tag{1}$$

Also $\theta_i > 0$ for all $i \in \{1, \ldots, m\}$, we get

$$\left\langle \sum_{i=1}^{m} \theta_i \xi_i, \eta(x, \bar{x}) \right\rangle < 0, \tag{2}$$

where $\sum_{i=1}^{m} \theta_i \xi_i \in \sum_{i=1}^{m} \theta_i \partial_c f_i(\bar{x})$.

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