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On the effect of demand randomness on inventory, pricing and profit

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ABSTRACT

We consider a stocking-factor-elasticity approach for pricing newsvendor facing multiplicative demand uncertainty with lost sales. For a class of iso-elastic demand curves, we prove that optimal order quantity decreases in demand uncertainty for zero salvage value. This contrasts with fixed-price newsvendor results which depend on the critical ratio. Numerical tests show that optimal order quantity increases in demand uncertainty for high salvage value, low marginal cost, and low price-elasticity. We also report results on optimal price, service level, and profit.

1. Introduction

Consider a pricing newsvendor facing multiplicative demand uncertainty with lost sales. We study the effect of demand randomness on the optimal price and order quantity, as well as on the optimal service level (i.e. normalized stocking factor). The impact of demand uncertainty on the firm's optimal decisions has been well studied, as summarized in Table 1. Gerchak and Mossman [2] first studied the fixed-price newsvendor with lost sales and found that while the optimal service level is independent of the demand uncertainty, optimal order quantity increases for high critical ratios and decreases for low critical ratios. For the pricing newsvendor, results depend on whether unsatisfied demand is lost or backlogged and demand uncertainty is multiplicative or additive. For lost sales under certain conditions, Li and Atkins [4] and Xu et al. [11] found that both optimal price and service level increase in demand variability for multiplicative demand uncertainty, whereas they both decrease in demand variability for additive demand uncertainty. Agrawal and Seshadri [1] considered backlogged demand satisfied by a more expensive emergency supplier. They found that under multiplicative demand uncertainty, optimal price is higher with uncertainty than without uncertainty while optimal order quantity is lower with uncertainty than without uncertainty. Under additive demand uncertainty, they found that optimal price and order quantity are independent of demand uncertainty. For both additive and multiplicative demand uncertainties, they also found that with demand uncertainty, optimal service level is lower for high critical ratios and higher for low critical ratios.

In recent years, elasticity-based approaches are gaining popularity in the study of the pricing newsvendor problem because demand elasticities are fundamental to the microeconomic analysis of pricing problems. Moreover, different elasticity approaches can be used to address different problems. For instance, Kocabıyıkoğlu and Popescu [3] show that the price-elasticity of lost-sales rate provides a general framework for establishing uniqueness of pricing newsvendor solutions. They also characterize how elasticity affects price and inventory, and vice versa. Another example is Salinger and Ampudia [9] who use price-elasticity of expected sales to generalize the Lerner relationship to price-setting newsvendors. This result provides a unified framework to understand the different effects of additive and multiplicative demand uncertainties. In this paper, we use both price-elasticity of demand and the stocking-factor-elasticity of expected sales used earlier in Petruzzi et al. [7].

Because of our focus on multiplicative demand with lost sales, our elasticity approach allows us to discover new relationships as well as closed forms for optimal decisions and profit of special cases. As summarized in Table 2, our contributions are as follows. For general demand curves, we discover a relationship between the price-elasticity of demand and the stocking-factor-elasticity of expected sales. We provide a simpler elasticity-based proof for the result that optimal price is increasing in demand uncertainty, and

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Table 1
Summary of the literature on the effect of demand uncertainty on optimal decisions.

Demand models			Price	Service level[a]	Order quantity
Fixed-price newsvendor [2]			N.A.	No change	↑ for high critical ratio, ↓ otherwise
Pricing newsvendor	Lost sales [4,11]	Multiplicative	↑	↑	N.A.
		Additive	↓	↓	N.A.
	Backlogged demand [1]	Multiplicative	↑	↓ for high critical ratio, ↑ otherwise[b]	↓[b]
		Additive	No change	↓ for high critical ratio, ↑ otherwise[b]	No change[b]

[a] Service level is defined as the normalized stocking factor.
[b] Results not explicitly claimed but inferred from the paper's results.

Table 2
Summary of our contributions on the effect of demand uncertainty on optimal decisions.

Demand model	Price	Service level	Order quantity
Multiplicative demand with lost sales	New proof	Generalized	New results

generalize Li and Atkins' [4] result for linear demand curve that the optimal service level is increasing in demand uncertainty. For a class of iso-elastic demand curves, we obtain the first explicit result for optimal order quantity of a pricing newsvendor with lost sales. We find that when salvage value is zero, optimal order quantity decreases in demand uncertainty. This result complements Agrawal and Seshadri's [1] result for backlogged demand. Moreover, this result holds even when the critical ratio is high, hence it contrasts with Gerchak and Mossman's [2] result for fixed-price newsvendor. Finally, numerical tests show that optimal order quantity increases in demand uncertainty when salvage value is high, marginal cost is low, and price-elasticity is low. These findings persist beyond iso-elastic demand curves, e.g. demand curve with linear form.

2. Model and results

Facing a random price-dependent demand, a firm's decision is to choose order quantity q and selling price p. We focus on the case where a change in price affects the scale of the demand distribution. In particular, uncertainty is incorporated into demand according to a multiplicative fashion as follows.

$$D(p, \xi) = y(p)\xi$$

where $y'(p) \leq 0$. An economic interpretation for this model is that ξ represents the uncertainty of the market size and $y(p)$ is the demand curve. See Petruzzi and Dada [6] and Li and Atkins [4] for more explanation on the validity of the model. We consider a general $y(p)$ by only assuming that it satisfies the property of increasing price-elasticity. Specifically, the *price-elasticity of demand* $\eta(p) = -py'(p)/y(p)$ is increasing in p. (Throughout this paper, we use increasing and decreasing in their weak sense.) This property is satisfied by various demand curves in the literature, including both the power (i.e. $y(p) = ap^{-b}$) and exponential (i.e. $y(p) = ae^{-p}$) forms in [6] and the linear (i.e. $y(p) = a - bp$) form in [4].

To study the effect of demand randomness, we consider a family of random variables

$$\xi_\beta = \beta\xi + (1-\beta)\mu$$

such that the mean and variance of ξ are μ and σ^2, respectively, and $0 \leq \beta \leq 1$. As β increases, the mean of ξ_β remains unchanged while the variance increases. For this reason, it is called the *mean-preserving transformation*, which is extensively used in microeconomics and is drawing increasing attention from the operations management community (e.g. [2,4]). Note that for any $\beta_1 \geq \beta_2$, ξ_{β_1} is *more variable* than ξ_{β_2} (see [2] for details), that is $\xi_{\beta_1} \geq_v \xi_{\beta_2}$. We let $f(x)$ (resp, $f_\beta(x)$), $F(x)$ (resp, $F_\beta(x)$) and $\bar{F}(x)$ (resp, $\bar{F}_\beta(x)$) be the probability density function, the cumulative distribution function and the complementary cumulative distribution function, respectively, for ξ (resp, ξ_β). For ease of exposition, we define the failure rate of ξ as $h(x) = f(x)/\bar{F}(x)$ and assume that ξ has increasing failure rate (IFR). This assumption is not restrictive as it is satisfied by a large range of probability distributions, including but not limited to the uniform, Weibull, normal, and exponential distributions, and their truncated versions. We further define the generalized failure rate of ξ_β as $g_\beta(x) = xf_\beta(x)/\bar{F}_\beta(x)$.

At the beginning of the selling season, the firm stocks q units of inventory at marginal cost c. At the end of the selling season, the leftover is salvaged at a unit value $s < c$. Given selling price p and market uncertainty ξ_β, the expected sales is $E\min\{q, y(p)\xi_\beta\}$ and the expected leftover is $q - E\min\{q, y(p)\xi_\beta\}$. Thus, the firm's expected profit is

$$\begin{aligned}\pi_\beta(p,q) &= pE\min\{q, y(p)\xi_\beta\} + s[q - E\min\{q, y(p)\xi_\beta\}] - cq\\ &= (p-s)E\min\{q, y(p)\xi_\beta\} - (c-s)q.\end{aligned}$$

For ease of analysis, we transform the decision variables from (p, q) to (p, z) where $z = \frac{q}{y(p)}$ is called the *stocking factor*. It follows that letting $S_\beta(z) = E\min\{z, \xi_\beta\}$,

$$\hat{\pi}_\beta(p,z) = (p-s)y(p)S_\beta(z) - (c-s)zy(p). \quad (1)$$

We denote the *stock-factor-elasticity of expected sales* as $\epsilon_\beta(z) = z\bar{F}_\beta(z)/S_\beta(z)$. Also, let the optimal decisions be p^*_β, q^*_β and z^*_β. The optimal profit will be $\pi^*_\beta = \hat{\pi}^*_\beta$. We now present our first result.

Lemma 1. *If ξ is IFR, then for any β,*

(a) $\epsilon'_\beta(z) < 0$,
(b) *There exists a unique solution (p^*_β, z^*_β) (equivalently, (p^*_β, q^*_β)) that satisfies*

$$\left[\frac{y(p)}{y'(p)} + (p-s)\right]S_\beta(z) = (c-s)z \quad (2)$$

$$\bar{F}_\beta(z) = \frac{c-s}{p-s}. \quad (3)$$

Moreover, price-elasticity of demand $\eta(p)$ and stocking-factor-elasticity of expected sales $\epsilon_\beta(z)$ are related as follows.

$$\frac{p}{p-s}\cdot\frac{1}{\eta(p)} + \epsilon_\beta\left(\bar{F}^{-1}_\beta\left(\frac{c-s}{p-s}\right)\right) = 1. \quad (4)$$