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Analyzing process flexibility: A distribution-free approach with partial expectations

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ABSTRACT

We develop a distribution-free model to evaluate the performance of process flexibility structures when only the mean and partial expectation of the demand are known. We characterize the worst-case demand distribution under general concave objective functions, and apply it to derive tight lower bounds for the performance of chaining structures under the balanced systems (systems with the same number of plants and products). We also derive a simple lower bound for chaining-like structures under unbalanced systems with different plant capacities.

1. Introduction

Product demand has become increasingly volatile, due to global market competition, product proliferation, and the enormous impact social media has on customer behavior. This calls for new production systems that can better cope with an increasingly volatile demand. As a result, process flexibility is quickly becoming an option that manufacturers embrace [11]. Interestingly, firms often do not need to implement a fully flexible system (also known as the full flexibility structure), where each plant has the ability to produce all products in the system [8]. Indeed, the seminal paper of Jordan and Graves [8] shows that in simulation, a sparse flexibility structure known as the long chain (also known as the chaining) often performs almost as well as the full flexibility structure.

The objective of this paper is to develop a new tool to analyze the performance of various process flexibility structures, and in particular, the popular chaining structure proposed by the seminal work of Jordan and Graves. Instead of taking the traditional approach of computing the expected sales of a flexibility structure under a given demand distribution, our paper takes a different approach and studies the worst expected sales of flexibility structures under a class of stochastic demand distributions with limited information. This is closely related to the distributionally robust literature, where one seeks to identify the optimal solution under the worst-case distribution within a distributional uncertainty set. Unlike most papers in the distributionally robust literature, where the set of distributions is defined by moment constraints, we consider a distributional uncertainty set with given partial expectations. By considering the distributional uncertainty set with given partial expectations, we explicitly characterize the worst-case demand distribution and using this characterization, we derive a simple analytical bound for the expected sales of chaining structures. Because the demand distribution is rarely known to a high degree of accuracy, our method enables us to evaluate the performance of flexibility structures in unbalanced and non-homogenous system where limited demand distributional information is known.

1.1. Literature review

The findings of [8] led to a series of researches to analytically study the effectiveness of the long chain and other sparse flexibility structures. [4] develops a method to compute the average demand satisfied by the chaining in asymptotically large systems; [5,13] analyze the chaining and other sparse flexibility structures under worst-case demand; [12] provides a characterization of the expected sales of the long chain and using the characterization,

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proves that the long chain always outperforms the shorter chains under i.i.d. demand; [3] uses probabilistic graph expanders to construct asymptotically optimal sparse structures; [6] analyzes the chaining with limited reserved capacities, and finally, [10] studies the problem of finding the optimal sparse flexibility configuration to achieve a given service level.

More closely related to this paper, [14] studies the k-chain (a structure where product i is capable of producing product $i, i+1, \ldots, i+k$) in asymptotically large balanced networks under i.i.d. demand using a distributionally-robust approach. The key difference between [14] and this work is that the former studies the worst-case demand distribution with given first and second moments, while this work studies the worst-case demand distribution with mean and partial expectations. The advantage of our approach is that we provide the exact characterization of the worst-case demand distribution for any finite flexibility structure, which allows us to develop a tool to study the broader class of non-homogenous unbalanced finite flexibility structures. In contrast, [14] does not fully characterize the worst-case distribution, and their closed-form lower-bound is restricted to symmetric, balanced systems with system size going to infinity. We note that while the characterization of the worst-case distribution with partial expectations was known since the 1970s (see [1]), this paper is the first to apply this idea to analyze process flexibility structures.

2. Model and assumptions

In this paper, we use $\mathbb{P}[\cdot]$ and $\mathbb{E}[\cdot]$ to denote the probability and the expectation functions of random variables. For two random variables D and D', we use $D \stackrel{d}{=} D'$ to denote that D and D' have the same probability distribution, i.e., $\mathbb{P}[D \le x] = \mathbb{P}[D' \le x]$ for all $x \in \mathbb{R}$.

We study a manufacturing system with n plants and m products, with $m \ge n$. For each $1 \le i \le n$, $1 \le j \le m$, c_i and D_j are used to denote the fixed capacity at plant i and the stochastic demand for product (type) j. A flexibility structure, denoted by $\mathscr{A}$, is a set of arcs connecting plant nodes to product nodes. In a flexibility structure $\mathscr{A}$, an arc $(i, j) \in \mathscr{A}$ implies that plant i is capable of producing product j. Given an instance $\mathbf{d}$ of the demand, the sales achieved by a flexibility structure $\mathscr{A}$, denoted by $S(\mathbf{d}, \mathscr{A})$, is defined as

$$\begin{aligned} S(\mathbf{d}, \mathscr{A}) := \max \quad & \sum_{(i,j)\in\mathscr{A}} f_{ij} \\ \text{s.t.} \quad & \sum_{i=1}^{n} f_{ij} \le d_j, \quad \forall 1 \le j \le m \\ & \sum_{j=1}^{n} f_{ij} \le c_i, \quad \forall 1 \le i \le n \\ & f_{ij} \ge 0, \quad \forall (i,j) \in \mathscr{A}. \end{aligned}$$

Under stochastic demand $\mathbf{D}$, the expected sales of $\mathscr{A}$ is hence denoted by $\mathbb{E}[S(\mathbf{D}, \mathscr{A})]$. Throughout the paper, we assume that the demand vector $\mathbf{D}$ is consisted of m independent random variables, and use μ_j to denote the expected values of D_j.

In the paper, we are interested in providing a lower bound for $\mathbb{E}[S(\mathbf{D}, \mathscr{A})]$ when the expected demand, e.g., μ_j, and the partial expectations of $D_j - \mu_j$ on interval $[0, \infty)$, e.g., $\mathbb{E}[(D_j - \mu_j)^+]$, are known. Note that

$$\mathbb{E}[(D_j - \mu_j)^+] = \mathbb{E}[(\mu_j - D_j)^+], \quad \text{and}$$
$$\mathbb{E}[(D_j - \mu_j)^+] + \mathbb{E}[(\mu_j - D_j)^+] = \mathbb{E}[|D_j - \mu_j|].$$

Therefore, $\mathbb{E}[(D_j - \mu_j)^+]$ is exactly one half of the expected absolute deviation of D_j from its mean. We say D_j is *γ-centralized* if $\mathbb{E}[(D_j - \mu_j)^+] \le \gamma\mu_j$. Clearly, if γ is small, then D_j has most of its probability measure to be concentrated around its mean. Like variance, the partial expectations under consideration, $\mathbb{E}[(D_j - \mu_j)^+]$, informs us about the degree of centralization of the demand.

3. Characterizing the worst-case distribution

In this section, we first characterizes the worst-case distribution which in turn bounds the expected values of general stochastic concave objective functions. Then, we apply this result to provide lower bounds for the expected sales of process flexibility structures.

Proposition 1. *Let $f(\cdot) : \mathbb{R}^m \to \mathbb{R}$ be an arbitrary concave function, and let $\mathbf{E}$ be an independent m-dimensional random vector where for all $1 \le j \le m$,*

$$\mathbb{P}[-\Delta_j^- \le E_j \le \Delta_j^+] = 1, \qquad \mathbb{E}[(-E_j)^+] = \gamma_j^- \Delta_j^-,$$
$$\mathbb{E}[(E_j)^+] = \gamma_j^+ \Delta_j^+,$$

where Δ_j^- and Δ_j^+ are positive reals. Then, we have that $\mathbb{E}[f(\mathbf{E}^*)] \le \mathbb{E}[f(\mathbf{E})]$, where $\mathbf{E}^*$ is an independent m-dimensional random vector such that

$$\mathbb{P}[E_j^* = -\Delta_j^-] = \gamma_j^-, \qquad \mathbb{P}[E_j^* = \Delta_j^+] = \gamma_j^+,$$
$$\mathbb{P}[E_j^* = 0] = 1 - \gamma_j^+ - \gamma_j^-, \quad \forall 1 \le j \le m.$$

The proof of Proposition 1 is a straightforward application of [9, 1] and is relegated to Appendix. Here, we describe the intuition behind the proof of Proposition 1. For each $1 \le j \le m$, we have partial expectations of E_j on intervals $[-\Delta_j^-, 0]$ and $[0, \Delta_j^+]$. Because the objective function f is concave, and E_j is independent with $E_{j'}$ for any $j' \ne j$, we can "transport" the probability of E_j on $[-\Delta_j^-, 0]$ and $[0, \Delta_j^+]$ to the points $\{-\Delta_j^-, 0, \Delta_j^+\}$ and obtain a valid independent distribution with a smaller expected objective value. After we do this for each j from 1 to m, we obtain $\mathbf{E}^*$, a distribution with smaller expected objective value than $\mathbf{E}$.

Recall that D_j is γ_j-centralized if $\mathbb{E}[(D_j - \mu_j)^+] \le \gamma_j\mu_j$. We next derive the result which allows us to characterize the distribution to lower-bound the expected sales of $\mathscr{A}$, when D_j is γ_j-centralized for each $1 \le j \le m$. Our derivation is done in two steps. In the first step, we show that $S(\mathbf{d}, \mathscr{A})$ is concave with respect to $\mathbf{d}$; in the second step, we apply Proposition 1 to obtain the worst-case distribution $\mathbf{D}^*$, for the set of all demand distributions where D_j is γ_j-centralized.

Lemma 1. *For any flexibility structure $\mathscr{A}$, $S(\mathbf{d}, \mathscr{A})$ is concave with respect to $\mathbf{d}$.*

Proof. Recall that $S(\mathbf{d}, \mathscr{A})$ is the objective of a linear program. Moreover, $S(\mathbf{d}, \mathscr{A})$ can be expressed as $S(\mathbf{d}, \mathscr{A}) = F(\mathbf{d}) = \max_{\mathbf{x}\in P(\mathbf{d})} \mathbf{c}^T\mathbf{x}$ for some vector $\mathbf{c}$, and some polyhedral $P(\mathbf{d}) = \{\mathbf{x}|\mathbf{A}\mathbf{x} \ge \mathbf{b}\}$. By Theorem 5.1 on pg. 213 of [2], $-F(\mathbf{d}) = \min_{\mathbf{x}\in P(\mathbf{d})} -\mathbf{c}^T\mathbf{x}$ is convex with respect to $\mathbf{d}$ and therefore, $S(\mathbf{d}, \mathscr{A}) = F(\mathbf{d})$ is concave with respect to $\mathbf{d}$. □

Proposition 2. *Let $\mathbf{D}$ be an m-dimensional independent demand vector where for $1 \le j \le m$, $\mathbb{E}[D_j] = \mu_j$, $\mathbb{P}[0 \le D_j \le \theta\mu_j] = 1$ with some $\theta > 1$ and D_j is γ_j-centralized, $\gamma_j \le \frac{\theta-1}{\theta}$. Then, for any flexibility structure $\mathscr{A}$, we have $\mathbb{E}[S(\mathbf{D}^*, \mathscr{A})] \le \mathbb{E}[S(\mathbf{D}, \mathscr{A})]$, where $\mathbf{D}^*$ is an m-dimensional independent demand vector such that*

$$\mathbb{P}[D_j^* = \theta\mu_j] = \frac{\gamma_j}{\theta - 1}, \qquad \mathbb{P}[D_j^* = 0] = \gamma_j,$$
$$\mathbb{P}[D_j^* = \mu_j] = 1 - \frac{\theta\gamma_j}{\theta - 1}, \quad \forall 1 \le j \le m.$$