Operations Research Letters 44 (2016) 403–408

Contents lists available at ScienceDirect

Operations Research Letters

journal homepage: www.elsevier.com/locate/orl

The two-level economic lot sizing problem with perishable items

Mehmet Önal *

Department of Industrial Engineering, Işık University, Şile, İstanbul, Turkiye

ARTICLE INFO

Article history:
Received 16 November 2015
Received in revised form
30 March 2016
Accepted 31 March 2016
Available online 8 April 2016

Keywords:
Lot-sizing
Dynamic programming
Perishable items
Complexity analysis

ABSTRACT

We present an economic lot sizing model of a supply chain for the procurement and distribution of a perishable item. We assume that the consumers always buy the item that lasts longer. We show that determining optimal procurement and transfer plan is $\mathcal{NP}$-hard, and present polynomial time solvable special cases.

© 2016 Elsevier B.V. All rights reserved.

1. Introduction

We consider a two level supply chain for the procurement and distribution of a perishable product. The upper (first) level represents the main storage location. Items procured from the supplier are initially stored in this location. They are transferred to the lower (second) level to satisfy demand. This setting represents a warehouse and a distribution center, where the distribution center satisfies the demands of the retailers which constitute the entire market (see e.g., [5]). We assume that consumers always buy the items that last longer. This leads to the so called LEFO (Last Expiration, First Out) consumption order.

Research on management of perishable items is vast. For a review of perishable inventory models, see [10], and [2]. In many of them, it is assumed that items are consumed in FIFO (First In; First Out) order. There are a few models that incorporate other consumption orders. Some examples include [7,12], who consider FIFO and LIFO (Last In; First Out) issuance policies, respectively, when the demand rate depends on the inventories. There are Economic Lot Sizing (ELS) models that account for perishability as well (see [4,6], and [3]). [9] are the first to analyze the effect of consumption order in a capacitated ELS model. They analyze FIFO, LIFO, FEFO (First Expiration; First Out) and LEFO (Last Expiration; First Out) orders. One of the main results of [9] is that, for any problem instance, lowest costs are achieved if the items are consumed in FEFO order, and highest costs are achieved if the items are consumed in LEFO order. The cost under any other consumption order is between these two extreme values.

As the foregoing discussion implies, costs can be reduced if the consumption order is manipulated. When the consumers have the power to choose, it is crucial to decide which items to present to the consumers together. If an early expiring item is together with a later expiring item, the early expiring item may never be sold and wastage may occur. In a supply chain, procurement and transfer decisions can be integrated to solve this problem. This idea motivates our research. We present a two level ELS model with perishable items (ELSPI-2L). Our model is an extension of the model in [9] where a separate storage location is introduced. It is also an extension of the two level model of [5], where perishability is incorporated.

The rest of the paper is organized as follows. In Section 2, we present our model. In Sections 3 and 4, we analyze the ELSPI-2L with, and without procurement bounds, respectively. We conclude the paper with Section 5.

2. Model formulation and related problems

2.1. The model

The ELSPI-2L is a two level ELS problem over a discrete planning horizon of T periods. The first level represents the warehouse and the second level represents the distribution center, where a total demand of D_t is satisfied in every period t ($t = 1, \ldots, T$). Items can be procured at the beginning of any period, and are sent to the warehouse. Procurement in period t cannot exceed the bound C, and it entails a set up cost of S_t, and a unit cost of p_t. Items are stored

* Tel.: +90 216 528 7138; fax: +90 216 712 1472.
E-mail address: onal@isikun.edu.tr.

http://dx.doi.org/10.1016/j.orl.2016.03.017
0167-6377/© 2016 Elsevier B.V. All rights reserved.

in the warehouse until they are transferred to the distribution center. Transfer in period t incurs a set up cost of R_t, and a unit cost of g_t. There are unit holding costs, h_t^1 and h_t^2, in period t, in the warehouse and in the distribution center, respectively. Items procured in period t expire on period $v_t \geq t$, and consumers always buy the item that has a later expiration date.

To formulate the ELSPI-2L, let x_{tji} be the amount of items procured in period t, transferred in period j and consumed in period i. Let I_t^1 and I_t^2 be the amount of inventory in period t in the warehouse and in the distribution center, respectively. Let x_t and u_t be the total procurement and transfer quantities in period t, respectively. Let y_t and w_t be variables that indicate set up in period t for procurement and transfer, respectively. Also, let the variable z_{tji} be equal to 1 if $x_{tji} > 0$, and 0 otherwise. Then, the ELSPI-2L (denoted by (P)) is formulated as follows.

$$\min \sum_{t=1}^{T} \left(S_t y_t + R_t w_t + p_t x_t + g_t u_t + h_t^1 I_t^1 + h_t^2 I_t^2\right)$$

subject to (P)

$$\sum_{t: t \leq i \leq v_t} \sum_{j: t \leq j \leq i} x_{tji} = D_i \qquad \text{for } i = 1, \ldots, T \tag{1}$$

$$x_t \leq C \qquad \text{for } t = 1, \ldots, T \tag{2}$$

$$x_{tji} \leq D_i z_{tji} \qquad \text{for } t = 1, \ldots, T;\ j = t, \ldots, v_t;\ i = j, \ldots, v_t \tag{3}$$

$$z_{tji} \leq y_t \qquad \text{for } t = 1, \ldots, T;\ j = t, \ldots, v_t;\ i = j, \ldots, v_t \tag{4}$$

$$z_{tji} \leq w_j \qquad \text{for } t = 1, \ldots, T;\ j = t, \ldots, v_t;\ i = j, \ldots, v_t \tag{5}$$

$$z_{ijs} + z_{\ell j' t} \leq 1 \qquad \text{for } 1 \leq i \leq j \leq s \leq v_i;\ 1 \leq \ell \leq j' \leq s < t \leq v_\ell \text{ with } v_i < v_\ell \tag{6}$$

$$\sum_{i=1}^{t} \sum_{j=t}^{T} x_{itj} = u_t \qquad \text{for } t = 1, \ldots, T \tag{7}$$

$$\sum_{j=t}^{v_t} \sum_{i=j}^{v_t} x_{tji} = x_t \qquad \text{for } t = 1, \ldots, T \tag{8}$$

$$\sum_{i=1}^{t} x_i - \sum_{i=1}^{t} u_i = I_t^1 \qquad \text{for } t = 1, \ldots, T \tag{9}$$

$$\sum_{i=1}^{t} u_i - \sum_{i=1}^{t} D_i = I_t^2 \qquad \text{for } t = 1, \ldots, T \tag{10}$$

$$x_{tji} \geq 0,\ z_{tji} \in \{0, 1\} \qquad \text{for } t = 1, \ldots, T;\ j = t, \ldots, v_t;\ i = j, \ldots, v_t$$

$$y_t,\ w_t \in [0, 1] \qquad \text{for } t = 1, \ldots, T.$$

The objective function minimizes total procurement, inventory holding, and transfer costs. Constraints (1) state that items cannot be allocated to satisfy periods beyond their expiration dates. Constraints (2) bound procurement in each period. Due to Constraints (3), $z_{tji} = 1$ if $x_{tji} > 0$, and 0 otherwise. If $z_{tji} = 1$, then $y_t = w_t = 1$ due to Constraints (4) and (5). Constraints (6) enforce LEFO consumption order.

Substituting equalities (9) and (10) in the objective function clears the inventory variables, and we get $\sum_{t=1}^{T}(S_t y_t + R_t w_t + p'_t x_t + g'_t u_t) + K$, where $p'_t = p_t + \sum_{j=t}^{T} h_j^1$, $g'_t = g_t + \sum_{j=t}^{T}(h_j^2 - h_j^1)$, and $K = \sum_{t=1}^{T} h_t^2 \sum_{j=t}^{T} D_j$. K is a constant and can be ignored. This implies that we can assume that holding costs are zero. Therefore, we assume $h_t^1 = h_t^2 = 0$ for $t = 1, \ldots, T$.

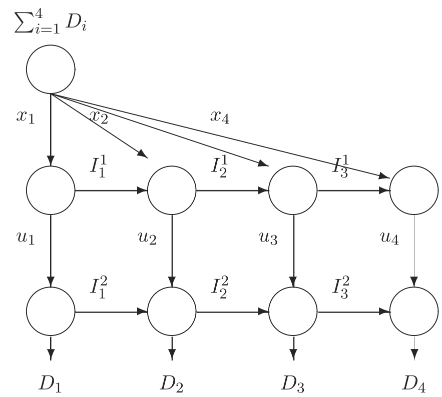

Fig. 1. Aggregate flow network representation of (P).

Depending on our assumptions on the lifetimes and the capacities, special cases of (P) arise. As far as the lifetimes are concerned, we have two cases. In one case, lifetimes are *non-overtaking*, such that $v_t \leq v_{t+1}$ for $t = 1, \ldots, T-1$. In the other case, lifetimes are *general*, such that we might have $v_t > v_{t+1}$ for some periods t as in [9]. We also investigate (P) with and without procurement bounds. We let P_C, P_U denote problem (P) with, and without procurement bounds, respectively. Likewise, we let P_{CN}, P_{UN} denote the special cases of P_C and P_U with non-overtaking lifetimes.

We call x_{tji}, the *allocation quantities*. They imply *aggregate quantities* x_t, u_t, I_t^1, and I_t^2 by constraints (7), (8), (9), (10). We call a period t a *procurement (transfer) period* if $x_t > 0$ ($u_t > 0$). We say procurement period t is fractional (full) if $x_t < C$ ($x_t = C$). Aggregate quantities correspond to a flow in a network with one source node, T transshipment and T demand nodes. Capacitated arcs connect the source node to transshipment nodes. The flow on these arcs is equal to the procurement in the corresponding periods. We call a transshipment node with a positive flow from the source node, a *procurement node*. There are arcs between transhipment and demand nodes. The flow on these arcs is equal to the amount of transfer in the corresponding periods. There are arcs between consecutive transhipment nodes and consecutive demand nodes. The flow on these arcs is equal to the inventory in the corresponding periods, in the warehouse and in the distribution center, respectively. We call this network, the *aggregate flow network*. A network for a 4 period problem is shown in Fig. 1.

2.2. Related problems

Multilevel ELS problems were studied by [8,5]. The study that is most closely related to ours is the one by [5], who present an $\mathcal{O}(T^7)$ algorithm to the two level ELS problem with concave cost functions and time-invariant procurement capacities. They reduce the complexity to $\mathcal{O}(T^5)$ when the holding and transfer cost functions are linear.

Our work is an extension of [9], who study the single level ELS problem with perishable items (ELS-PI) under LEFO, FEFO, FIFO and LIFO consumption orders. We denote the ELS-PI by (P^{1L}). To refer to the variables (or parameters), we drop the superscripts of the variables of (P) (e.g., h_t is the unit holding cost, I_t is the inventory in period t). We let P_U^{1L}(M) and P_C^{1L}(M) denote problem P^{1L} with no, and time-invariant procurement bounds, respectively, under a consumption order M $\in$ {FEFO, LEFO, FIFO, LIFO}. [9] show that P_C^{1L}(FEFO) and P_C^{1L}(LIFO) are $\mathcal{NP}$-hard, while P_C^{1L}(FIFO), P_C^{1L}(LEFO) can be solved in $\mathcal{O}(T^4)$ time. [9] propose $\mathcal{O}(T^4)$, $\mathcal{O}(T^3)$, $\mathcal{O}(T^2)$ and $\mathcal{O}(T^2)$ algorithms for P_U^{1L}(FEFO), P_U^{1L}(LIFO), P_U^{1L}(FIFO), and P_U^{1L}(LEFO), respectively.

Download English Version:

https://daneshyari.com/en/article/1142150

Download Persian Version:

https://daneshyari.com/article/1142150

Daneshyari.com