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Quantity strategies in economic networks

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ABSTRACT

We study the optimal quantity strategies of the firm which owns many subsidiaries embedded in an economic network. A key feature of our model is that subsidiaries experience a negative local network effect. First, we show that there exists a unique Nash equilibrium in the game. Second, we characterize the equilibrium strategies by considering some specific network structures. Then, we identify how changes in the payoff parameters affect equilibrium play. Finally, we also analyze the strategy features of different models through two simple examples.

1. Introduction

Inarguably, economic networks that describe the pattern and level of interaction of a set of agents are instrumental in the effects of collaboration and the modes of competition among its members. The most notable example is that of the market for fast food, which requires a plenty of restaurants in the form of a distribution network. Then, the market power of firms is determined by the market structure and the network structure. Nowadays the great majority of restaurants who are embedded in a network are owned only by a few firms. For example, in the fast food restaurant chain sector, leading firms such as KFC and McDonalds control thousands of restaurants all over the world. Also, multinational retailer Wal-Mart maintains high store density and a contiguous store network all along the way. The major challenge that arises naturally is whether firms can intelligently use the network structure to improve their business strategies.

In this paper, we study the Cournot game based on the economic network, where each node represents a subsidiary, and the firm chooses the quantity levels of its subsidiaries. A main feature of the subsidiaries we consider is that they exhibit a local network effect: increasing the quantity level of a subsidiary has a negative impact on the output levels of her neighbors, i.e., games are strategic substitutes. We also assume that firms have linear demand function and then their profit functional form is linear–quadratic, which enables us to obtain structural insights on the optimal strategies.

First, in an oligopolistic market, we show that the game of strategic substitutes has a unique Nash equilibrium, while Solan and Vieille [20] study equilibrium uniqueness with perfect complements. We obtain the equilibrium conditions and provide an interpretation of the equilibrium quantities using the Katz–Bonacich centrality introduced by Katz [16] and Bonacich [3], which reveals the strategic interactions between links. Next, we show the importance of the network structure to economic outcomes by considering some specific network structures like bipartite networks. Then, we study how changes in the payoff function parameters affect equilibrium play. Strategic substitutes pose a challenge for comparative statics, but this paper shows that an increase in the payoff parameters lowers the equilibrium play. Finally, we also analyze the strategy features of different models and obtain some insights through two simple examples.

We now discuss briefly the related literature. Models of local network externality which explicitly take into account game theory have been proposed by Ballester et al. [1], Bramoullé and Kranton [4,5], Corbo et al. [8], and Galeotti and Goyal [10]. A key modeling assumption in the above models, which we also adopt in our setting, is that the best reply functions are linear. Ballester et al. [1] are the first to note the linkage between Bonacich centrality and Nash equilibrium outcomes in a single-stage game with local payoff complementarities. On the contrary, we study strategic interactions with a focus on games of strategic substitutes.

Recently, there is a stream of literature that studies a set of pricing questions related to marketing strategies over social networks.

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See, e.g., Manshadi and Johari [17] for Bertrand competition; Candogan, Bimpikis and Ozdaglar [7], Bloch and Quérou [2] for monopoly pricing. Ghasemieh et al. [11] consider a market in which two competing sellers offer two similar products in a social network. Also, there is a growing literature on industrial organization and networks, much of which has focused on network formation. See, e.g., Goyal and Moraga [14], Goyal and Joshi [12], Deroian and Gannon [9], Goyal, Konovalov and Moraga-Gonzalez [13], and Westbrock [21] for Cournot competition and R&D networks. But their analysis highlights the architecture of collaboration networks.

Given a set of quantities, our model takes the form of a network game among nodes that interact locally. Bulow et al. [6] are the earliest example of a Cournot analysis in a network of markets and firms. Nava [19] studies quantity competition in a network of Walrasian agents who simultaneously buy and sell. He provides conditions for the existence of an equilibrium both when sellers make the offers and when buyers make the offers. Ilkilic [15] models a bipartite network where links connect firms with markets and looks at the Cournot game in which firms decide how much to sell at each market they are connected to. However, in our model, nodes interact directly only with a subset of nodes. More recently, another related model to ours is a work by Bramoullé, Kranton and D'Amours [5] who study Cournot competition in the network and use the theory of potential games (Monderer & Shapley [18]) to analyze the Nash equilibria. Equilibria depend on a single network measure: the lowest eigenvalue. Their paper is the first to uncover the importance of the lowest eigenvalue to economic and social outcomes. Nevertheless, they assume that each node represents a firm, which is considerably different from ours.

The paper proceeds as follows. The next section presents the basic model. Section 3 shows that there exists a unique Nash equilibrium in the game. In Section 4, we first relate the equilibrium strategies to network structure, and then conduct comparative statics. Section 5 compares the two settings. Finally, we conclude in Section 6.

2. The model

There are m $(2 \leq m \leq n)$ firms in a market which consists of a set $N = \{1, 2, \ldots, n\}$ of nodes embedded in an economic network, and each firm possesses at least one node in the network. That is, firm's subsidiaries are known as nodes in the network. Each firm i controls a subset of nodes N_i, and $|N_i| = n_i$, $i = 1, \ldots, m$, such that $N_1 \cup N_2 \cup \cdots \cup N_m = N$ and $N_1 \cap N_2 \cap \cdots \cap N_m = \varnothing$. Without loss of generality, N_1 is made up of the former n_1 nodes, and so on. Moreover, this network is represented by the adjacency matrix G. The jk-th entry of G, denoted by g_{jk}, represents the strength of the influence of node k on j. For any pair of nodes $j \neq k$, $g_{jk} > 0$ if there exists an edge between j and k, and $g_{jk} = 0$ otherwise. And we normalize $g_{jj} = 0$ for all j. Moreover, we assume that the network is undirected, $g_{jk} = g_{kj}$, so that G is a symmetric matrix.

Firm i introduces a divisible good in the market and chooses a vector of quantities $\mathbf{q}_i = [q_j]_{j\in N_i}$ from the set of allowable strategies, where $q_j \geq 0$ is the quantity of subsidiary j. We assume that there is no fixed cost of production and the marginal cost is constant at c. Then, the firm i's profit function can be given by an expression of the following form:

$$\pi_i = \sum_{j\in N_i} q_j(p_j - c), \tag{1}$$

where the price p_j of node j chosen by firm i is given by

$$p_j = a_j - \frac{1}{2} b_j q_j - \sum_{k\in N} g_{jk} q_k, \quad j = 1, \ldots, n.$$

The model parameter $a_j > 0$ represents the market capacity of subsidiary j, and the model parameter $b_j > 0$ denotes the influence of quantity of subsidiary j on its price. The third term represents the negative network effect of its economic group. We next set $b_j = b$ for all $j = 1, \ldots, n$.

Then, we make the following assumption that ensures that it is profitable for the duopolist to choose a positive quantity in the absence of the network effects.

Assumption 1. For all $j \in N$, $a_j > c$.

Assumption 1 guarantees all the firms gain a positive amount of the profit, i.e., the equilibrium features an interior quantity vector. This enables us to state our results in a closed form and draw explicitly the connection between the optimal quantity strategy of the firm and a measure of network influence.

Finally, we assume that firms simultaneously choose their quantity vectors so as to maximize their profits.

3. Nash equilibria

In this section, we study the game defined in Section 2 under Assumption 1 and characterize the Nash equilibria of the game among the firms. In particular, we show that the equilibrium is unique.

Let $r_j(\mathbf{q}_{-j})$ denote the best response of subsidiary j owned by firm i, where $\mathbf{q}_{-j}$ denotes the quantity levels of all nodes but j. From (1), it follows that:

$$r_j(\mathbf{q}_{-j}) = \frac{1}{b_j}\left[a_j - c - \sum_{k\in N_i} g_{kj} q_k - \sum_{k\in N} g_{jk} q_k\right]. \tag{2}$$

Thus, expression (2) can be interpreted in the following manner. First, note that if each node is a firm and maximizes its profit, the best response for node j would be $\widetilde{r}_j(\mathbf{q}_{-j}) = \frac{1}{b_j}[a_j - c - \sum_{k\in N} g_{jk} q_k]$. As a result, $r_j(\mathbf{q}_{-j})$ is equal to $\widetilde{r}_j(\mathbf{q}_{-j})$, adjusted downward by the factor $\frac{1}{b_j}\sum_{k\in N_i} g_{kj} q_k$. This adjustment indicates the network effect of its neighbors who belong to the same firm, which implies that the firm internalizes the externality. Finally, the term $\sum_{k\in N} g_{jk} q_k$ generates the network effect. From the expression (2), a_j is greater than c, and without loss of generality, we consider from now on prices net of marginal cost.

To express results in a compact form, we define the vectors $\mathbf{a}, \mathbf{q}, \mathbf{1} \in \mathbb{R}^n$, such that $\mathbf{a} = [a_j]_j$, $\mathbf{q} = [q_j]_j$, $\mathbf{1} = [1_j]_j$, and define an identity matrix $I \in \mathbb{R}^{n\times n}$. In addition, we also define a matrix $\bar{G} \in \mathbb{R}^{n\times n}$, where $\bar{g}_{jk} = g_{kj}$ if and only if j and k are controlled by the same firm, and $\bar{g}_{jk} = 0$, otherwise. Because matrix G is symmetric, $\bar{G}$ is also symmetric. By Eq. (2), it follows that the equilibrium condition of the game is given by

$$bI\mathbf{q}^* = \mathbf{a} - G\mathbf{q}^* - \bar{G}\mathbf{q}^*. \tag{3}$$

When $m = 2$, the block matrix G_i shows the connection of nodes controlled by firm i, $i = 1, 2$. On the other hand, G_3 and G_4 represent the links between nodes of firms 1 and 2. Accordingly, I_1 and I_2 stand for the block matrices of identity matrix I. Then, the block matrices of G and I can be expressed as

$$G = \left(\begin{array}{c|c} G_1 & G_3 \\ \hline G_4 & G_2 \end{array}\right), \qquad I = \left(\begin{array}{c|c} I_1 & 0 \\ \hline 0 & I_2 \end{array}\right).$$

According to the network structure, let $\mathbf{a} = (\mathbf{a}_1^T, \mathbf{a}_2^T)^T$, where $\mathbf{a}_i$ represents the vector of firm i, $i = 1, 2$, and $\mathbf{q} = (\mathbf{q}_1^T, \mathbf{q}_2^T)^T$ similarly.

Definition 1. For a network with adjacency matrix G and a scalar α, the vector of Katz–Bonacich centrality of parameter α is $\mathbf{w}(G, \alpha)$