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Optimal investment strategy under time-inconsistent preferences and high-water mark contract

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ABSTRACT

This paper considers the optimal investment problem for a fund manager who has time-inconsistent preferences and is compensated with a HWM contract. The time preferences of fund manager is described by the stochastic hyperbolic discounting function. The closed-form solution under certain conditions is provided by applying the dynamic programming approach. Interestingly, we find that the sophisticated fund manager is present-biased. The more the fund manager has present-biased preference, there is the greater inclination to increase the proportion in risky asset.

1. Introduction

Most investors delegate the management of a fraction of their wealth to fund managers. Hedge funds are a key group of financial intermediaries. In US, hedge funds had more than $1.47 trillion in net assets (i.e., equity) in 2013, similar to the total equity of US banks, and more than one trillion dollars in borrowing. In the hedge fund industry, fund managers achieve regular management fees and performance incentive fees. The most common schedule, the regular management fees are paid as annual fees of 2% of assets and performance incentive fees are paid according to a high-water mark (HWM) contract. That is, the performance incentive fees are paid only when the fund exceeds its previous high-water mark.

Under the HWM contract setting, quantitative time-continuous valuation framework for management and performance incentive fees is firstly resolved in [10]. Later, [24] investigates the portfolio choice of hedge fund managers under HWM contract and first provides explicit result for the manager's investment optimization problem and show that the fund manager optimally acts like a CRRA investor with a fixed risk-aversion of less than one. Similar to [24], [8] investigates the risk-choice problem for a risk-neutral manager who faces a HWM contract. In addition, [17] studies the economics of hedge funds where the fund managers are compensated by HWM contract. They assume that the fund manager allocates the fund wealth between the alpha-generating strategy and the risk-free asset. By leveraging the alpha strategy, the manager creates value for investors (inexpectation) and hence benefits via performance-linked compensation.

In the above-mentioned literature, it is assumed that the fund manager has a constant rate of time preference. Thus the models typically assume that the performance fees are discounted exponentially. As we well know, such preferences are time-consistent in that a fund manager's preference for her performance fees is the same at an earlier date over a later date no matter when he is rewarded. However, actually every experimental study about time preferences suggests that the assumption of time-consistency is unrealistic. In psychology and behavioral science, there exists overwhelming evidence that has shown that time-inconsistency is standard in human preferences. For instance, [2,3,7,15,18,20,21,26] and reference therein. In fact, there is substantial evidence indicating that the discount rate is a decreasing function of time, which means that people are impatient about choices in the short term but are patient for long-term alternatives. That is, during pursuing immediate gratification, one often exhibits a reversal of preferences when choosing between a smaller, earlier reward and an alternative larger, but later reward. [16] models such time-varying

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impatience as hyperbolic discount functions, in which the discount rate decreases as the horizon increases. Such preferences are also called "present-biased" preferences by [22]. The hyperbolic discounting model has become the most widely accepted framework for modeling time-inconsistent preferences in economics. A huge literature has been developed to address a wide range of issues in economics based on hyperbolic discounting, which includes [11,22], and [23], among others.

It is a remarkable fact that the optimization problems with a hyperbolic discounting discount function are time-inconsistent, in the sense that the value functions lack the iterated-expected property and that Bellman's optimality principle does not hold (see [5]). Thus the dynamic programming approach cannot be directly adopted to solve such optimization problems. [25] suggests three ways to deal with the optimization problems with time-inconsistent preferences: (i) adopting some technologies (for instance, signing a contract in advance) so that the decision maker's future behaviors are irrevocable, (ii) assuming the decision maker to be naive and ignoring the conflict as a spendthrifty, or (iii) assuming the decision maker to be sophisticated and considering a strategy of consistent planning. Ways (i) and (ii) lead to the time-inconsistent strategies, i.e., in the sense that the optimal strategy made at a moment is NOT necessarily optimal at a later time. Whereas, during making decisions, way (iii) imposes that the decision maker should take into account her future actions induced by her time-varying preferences. Adopting way (iii), the strategies are time-consistent. Generally, under time-inconsistent preferences, way (iii) are realized by taking the game theoretic point of view and considering so called subgame perfect Nash equilibrium strategies. [25] firstly analyzes the behaviors of a decision maker with time-inconsistent preferences. Later, [14] applies Strotz's ideas in the study of environmental regulation, [11] studies real options in the framework of time-inconsistent preferences, [9] applies the framework of time-inconsistent preferences in the fishery resource management problem, [12,27] investigate the classical consumption problem with time-inconsistent preferences. [6] investigates the optimal dividend strategies when the manager has time-inconsistent preferences.

The present paper investigates the investment optimization problem for a fund manager with time-inconsistent preferences. To the best of our knowledge, we are the first to investigate the time-inconsistent optimization problem for a fund manager. For simplicity, assume that the fund manager can invest her fund wealth in a financial market consisting of one risky asset and one risk-free asset, meanwhile she is rewarded the performance incentive fees according to HMW contract. Following with [11–13], time-inconsistent preferences of the fund manager are described by a continuous-time version of the quasi-hyperbolic discounting function. As in standard literature on time-inconsistent behaviors, the fund manager with inconsistent time preference is modeled as a sequence of autonomous selves. Each self makes investment decision during her own present and cares about but does not control—investment decision in her future. Our problem is therefore an intrapersonal game. [19] defines Markov-perfect equilibrium (or MPE for short). MPE is a refinement of subgame-perfect equilibrium which only allows strategies to depend on information that is directly payoff relevant (i.e. information that is necessary to determine players' choice sets or payoffs). It does not allow strategies to depend on information that is only indirectly relevant (e.g. it does not allow the strategy of one player to depend on information that only becomes relevant if the strategy of another player depends on it). We go further, restricting attention to stationary MPE. Following the literature in intergenerational games, our solution concept for this game will be stationary MPE strategy.

Assume that the fund manager is sophisticated, we consider the optimization problem for a fund manager with time-inconsistent preferences. Her current self correctly foresees the preference difference of future selves, and future selves act according to the preferences of the current self. The optimization problem is firstly translated into standard singular control problem, and then the HJB equation is derived by applying the dynamic programming approach. The solutions are be derived in closed-form, which provide us an opportunity to observe the sensitivity of the optimal strategies to model parameters. Interestingly, our results show that the investment strategy under the time-inconsistent preferences is larger than that of a time-consistent fund manager. Moreover, we rigorously illustrate that a fund manager with time-inconsistent preferences will become present-biased. That is, the fund manager would prefer to invest more in the risky asset so as to obtain the performance incentive fee early.

The rest of the paper is organized as follows. Section 2 introduces the model setup and the time preferences. Section 3 provides HJB equation and the optimal strategy for sophisticated manager with stochastic hyperbolic discounting. Section 4 compares the dynamic behaviors of fund managers with stochastic hyperbolic discounting and exponential discounting. We present technical proofs in Appendix online (see Appendix A).

2. Assumption and model

Let $(\Omega, \mathscr{F}, \mathbb{P})$ be a probability space equipped with a filtration $\mathbb{F} = (\mathscr{F}_t)_{0\le t\le T}$ satisfying the usual conditions, i.e., $(\mathscr{F}_t)_{0\le t\le T}$ is right-continuous and $\mathbb{P}$-complete, where $T > 0$ is the time horizon. Suppose $\{W(t),\ t \ge 0\}$ be a standard one-dimension standard Brownian motion defined on the filtered probability space $(\Omega, \mathscr{F}, \mathbb{F}, \mathbb{P})$.

2.1. Model setup

Assume that the fund manager has two investment opportunities in a financial market. The first is a risk-free asset B_t in the money market which pays a rate of return r with certainty. The second is a risky asset S_t whose price evolves as a geometric Brownian motion,

$$dS_t = \mu S_t dt + \sigma S_t dW_t, \qquad S_0 = s_0 \ge 0, \tag{2.1}$$

where $\mu > r$ and $\sigma > 0$ denote the mean and volatility of the asset return respectively, W_t is a one-dimensional standard Brownian motion. Here, $\mu > r$ implies that the fund manager perceives that there exist some investment opportunities in the market which will yield more than r.

Let X_t be the fund wealth which is managed by the fund manager at time t. During investment period, the manager dynamically invests proportion of her fund wealth π_t in the risky asset at time t; the remainder of fund wealth, $(1-\pi_t)X_t$, is invested in the risk-free asset. The stochastic process $\{\pi_t\}_{t\in[0,T]}$ is called an investment strategy. Note that we allow short selling and borrowing/lending at the risk-free rate.

As in GIR, fund investors are allowed to continuously *redeem/withdraw* capital at constant rate δ, i.e., capital outflow induced by investors' withdrawal is δX_t at time t. Meanwhile, the withdrawals lead to certain expenses, denoted by $c'X_t$. We call c' the cost rate of withdrawals. Note that $0 < c' < 1$, $0 < \delta < 1$.

The fund manager accepts the fund management case by some contract in which she is rewarded by both management fee and performance incentive fees. Generally, the management fee is paid by a constant percent c of the fund wealth X_t, e.g., 2%. Assume that $c \le c'$. That is to say that earlier redemption is not cheap. The performance incentive fees usually directly link to the fund's performance, in this paper we assume that the performance incentive fees is paid on basis of the HWM contract.