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Forward-buying and the naive newsvendor

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ABSTRACT

The newsvendor model assumes that demand in a period is independent of the discounted inventory in the previous period. In the presence of forward-buying consumers, discounted inventory in a period may reduce next period's demand. We find that incorporating forward-buying leads to smaller order quantities and possibly lower salvage value.

1. Introduction and literature review

The classic newsvendor problem is to find a product's order quantity that maximizes the expected profit under probabilistic demand. The newsvendor model assumes that if any inventory remains at the end of the period, a discount is used to sell it or it is disposed of [7]. If the order quantity is smaller than demand, the newsvendor forgoes some profit. The newsvendor model is reflective of many real life situations and is often used to aid decision making [11].

An essential assumption in the newsvendor model is that excess inventory is discounted at the end of the period and the demand in the subsequent period is independent of the quantity of discounted inventory at the end of the current period. This makes the demand in each period independent of the demand and order quantity of the previous period.

The above model is valid only for products which we define as two-sided products. A product is two-sided if it loses significant value to both the newsvendor and consumers at the end of the period, e.g., real Christmas trees and newspapers. At the end of the day, a newspaper loses its value for both the newsvendor and the consumer. On the other hand, a one-sided product losses much more value to the newsvendor than to consumers, e.g., artificial Christmas trees. An artificial Christmas tree is bought by consumers for repeated use over several periods and, therefore, maintains a higher value to consumers than to the newsvendor, who must clear inventory at period's end. Many products which are treated in the literature as two-sided are in reality one-sided. The newsvendor must clear inventory of these products because next period's products may have different styles, e.g., winter jackets, may require large storage space, e.g., artificial Christmas trees, or may have better performance, e.g., USB flash drives [3]. These products include seasonal sports and fashion products and some information technology products. These products are purchased by consumers for repeated use over more than one period.

The distinction between one-sided vs. two-sided products is more important in the presence of strategic forward-looking consumers. When strategic consumers make up a large segment of all consumers, the assumption that demand in a given period is independent of the order quantity and excess inventory in the previous period is not valid. Forward-looking consumers will forward-buy many products when a newsvendor offers a large discount at the end of the period to avoid paying full price in the following period. We refer to a newsvendor who sells a one-sided product and ignores the forward-buying behavior of consumers as a naive newsvendor.

The effect of strategic consumers on retailers' pricing and profitability has been the focus of much recent research. Cachon and Swinney [2] contributed to our understanding of the effect of strategic consumers on inventory and pricing policy of retailers and on their profits. In their model, strategic consumers decide between purchasing the product at the regular price or waiting to purchase later, but still before the salvaging stage. Levin et al. [9] distinguished strategic consumers from others by their awareness that pricing is dynamic and by timing their purchases accordingly. Strategic consumers are patient and can weigh the benefit of delaying or accelerating purchases [1]. In this paper, some strategic consumers, who have intentions to buy a product in the next

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period, may, upon finding the product discounted at the end of the current period, accelerate their purchase and buy the product and consequently are no longer part of the demand in the next period. For example, a consumer who finds that her artificial Christmas tree is getting old as she puts it up may decide to finish this period with it and buy a new tree next holiday season. This consumer may buy a discounted tree if she finds one at the end of the current period with a large enough discount and discards the old tree. If a large number of such consumers can find discounted trees, then next period's demand is no longer independent of the current period's excess inventory. This will violate one of the essential assumptions of the newsvendor model.

To analyze the above problem, we divide consumers into three segments, bargain-hunting, myopic, and forward-looking consumers [2]. Bargain-hunting consumers only buy deeply discounted products. These consumers visit the retailer only if they know that the price has been deeply discounted and they do not buy products unless they believe they are getting a "deal". Bargain-hunters buy the product only in the post-season of a period. Myopic consumers buy the product in the period they will use it, i.e. they do not anticipate their future needs. Forward-looking consumers anticipate their future needs and will forward-buy a product if the discount is large. Khouja [8] analyzed the effects of large order quantities in the newsvendor problem under discrete demand without consideration of bargain-hunting consumers. The model also assumed a fixed salvage value and a fixed order quantity per period as long as demand has the same distribution.

To examine the impact of forward-looking consumers on the optimal order quantity and profit, we consider a two-period newsvendor problem. While the newsvendor model is typically addressed in a single period, multi-period models have been developed in the literature [6]. We analyze the case in which the number of bargain-hunters is constant, i.e. independent of the product salvage value (discounted price). We then extend the model to treat the salvage value as endogenous and reduced salvage value is used by the newsvendor to attract more bargain-hunters. We assume a risk-neutral newsvendor while noting that other attitudes toward risk have been used in the literature [4]. We perform sensitivity analysis using a numerical example.

2. Two-period model with forward-buying

Consider a newsvendor deciding on the quantity to order and define the following notation:

$i = 1, 2$ a period index,
x_i = demand in period i, x_i are independent and identically distributed,
$f(x_i)$ = the probability density function of x_i,
$F(x_i)$ = the cumulative distribution function of x_i,
μ = the mean of x_i,
σ = the standard deviation of x_i,
Q_i = the order quantity in period i,
P = selling price per unit,
C = cost per unit,
V_c = salvage value per unit,
B_o = number of bargain-hunters who will buy the product at price V_c,
ρ = proportion of forward-looking consumers in the consumer population, and
r = a discount factor which applies to second period's profit.

Ignoring forward-buying, the optimal order quantity of a naive newsvendor is given by the well-known fractile formula [7]

$$F(Q_c^*) = \frac{P - C}{P - V_c}. \tag{1}$$

Now suppose a fraction of ρ consumers are forward-looking. During period 1, these consumers realize that they will need to replace their product next period, i.e. they will be part of period 2's demand. These consumers will buy the product during the post-season of period 1 if it is available at a reasonable discount. The discount must be large enough to compensate them for their holding cost. If $P - V_c > \textit{consumer holding cost}$, which we assume to be true since the salvage value in the classic newsvendor is considerably smaller than the regular price, then forward-looking consumers will purchase the product in the post-season of period 1. Many factors may still effect the realization of demand in period 2. Thus, we use $\rho\mu$ as an estimate of forward-looking consumers from period 2 who will buy the product in the post-season of period 1 for \$$V_c$ per unit, if it is available. The actual demand from period 2 which shifts to the post-season of period 1 depends on the relative sizes of the bargain-hunter and forward-looking consumer segments. Since there are $\rho\mu$ expected forward-looking consumers in period 2 and B_o bargain-hunters who want to buy the product in the post-season of period 1, they form a queue for period 1's excess inventory in which every $\delta = \frac{\rho\mu}{\rho\mu + B_o}$ is a forward-looking consumer [2]. The amount of excess inventory available in the post-season of period 1 to both bargain-hunters and forward-looking consumers is $(Q_1 - x_1)^+$. Based on the above assumption, we have two cases:

1. $(Q_1 - x_1)^+ = 0$, no demand shifts from period 2 to the post-season of period 1 since there is no excess inventory in period 1. Demand in period 2 is unaffected by forward-looking consumers and is given by x_2, or
2. $(Q_1 - x_1)^+ > 0$, $\delta(Q_1 - x_1)$ will be demanded by forward-looking consumers. This will result in one of two outcomes:
   2.a. $0 < \delta(Q_1 - x_1)^+ < \rho\mu$, excess inventory in period 1 is insufficient to meet all forward-buying consumers' demand. The demand in period 2 becomes $y_1 = x_2 - \delta(Q_1 - x_1)^+$.
   2.b. $\delta(Q_1 - x_1)^+ > \rho\mu$, excess inventory in period 1 is sufficient to meet all forward-buying consumers' demand. The demand in period 2 becomes $y_2 = x_2 - \rho\mu$.

Case (2.b) implies that $\frac{\rho\mu}{\rho\mu + B_o}(Q_1 - x_1)^+ > \rho\mu$ which can be rewritten as $(Q_1 - x_1)^+ > \rho\mu + B_o$. For reasonable values of the size of the bargain-hunting consumer segment and reasonably large ρ, $Pr(Q_1 - x_1 > \rho\mu + B_o) \approx 0$ and therefore, for simplicity, we do not consider this case. The profit in the first period is

$$Z_1 = \int_{Q_1}^{\infty} Q_1(P - C)f(x_1)\,dx_1 + \int_{-\infty}^{Q_1} x_1(P - C)f(x_1)\,dx_1 + \int_{-\infty}^{Q_1} (V_c - C)(Q_1 - x_1)f(x_1)\,dx_1. \tag{2}$$

Since the newsvendor will know if $Q_1 - x_1 > 0$ prior to making the second period's order quantity decision, she will use that information in making the decision. Therefore, the second period's quantity decision will be made optimally. Let $\hat{x}_1$ be the realized value of x_1 at the end of period 1. If $Q_1 - \hat{x}_1 < 0$, then the demand in the second period is independent of the first period's demand. Let $Q_{2,1}$ be the second period's order quantity if $Q_1 - \hat{x}_1 < 0$, the second period's profit, denoted by $Z_{2,1}$, is

$$Z_{2,1} = \int_{Q_{2,1}}^{\infty} Q_{2,1}(P - C)f(x_2)\,dx_2 + \int_{-\infty}^{Q_{2,1}} x_2(P - C)f(x_2)\,dx_2 + \int_{-\infty}^{Q_{2,1}} (V_c - C)(Q_{2,1} - x_2)f(x_2)\,dx_2, \tag{3}$$