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An extension of the stochastic Joint-Replenishment Problem under the class of cyclic policies

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ABSTRACT

This paper presents an extension of the Joint-Replenishment Problem under the class of cyclic policies. The developed model includes stochastic demands, backorders-lost sales mixtures, and controllable lead times. With the objective of minimizing total system cost, we propose two heuristics. Numerical experiments investigate the algorithms performance and the model sensitivity.

1. Introduction

Some multi-product systems may require coordinated replenishments [13]. The related optimization problem is known as the Joint-Replenishment Problem (JRP), which is characterized by a cost structure that permits achieving substantial savings thanks to group replenishments [9,10].

Although the JRP literature is extensive (see [9] for the latest review and Table 1 for an overview of more recent papers), controllable lead times and backorders-lost sales mixtures have never been introduced, to the best of our knowledge, in the JRP under the class of cyclic policies. These aspects are relevant in inventory management [8,22,11], though.

This paper considers, therefore, the periodic-review JRP under the class of cyclic policies, taking into account stochastic demands, backorders-lost sales mixtures, and controllable lead times. Note that the periodic-review policy is widely spread in practice, being structurally simple and easy to implement [21]. Moreover, it seems preferable to the continuous-review policy in the JRP framework [7].

The objective is to determine the cyclic replenishment policy and the length of lead times that minimize the expected total cost per time unit. The simplest deterministic JRP is NP-hard [3], and the exact optimization may be computationally prohibitive for large problems [9]. Since our problem is at least equally complex to be solved exactly (it is a generalization of the basic JRP formulation), a heuristic method is thus needed.

We first present an effective heuristic algorithm, which may turn out to be computationally onerous for large problems. Therefore, we then propose a more efficient solution procedure. We finally examine the performances of the developed algorithms and the sensitivity of the model with numerical experiments.

2. Preliminaries

A family of items is procured from one vendor. Items are managed under a periodic-review policy with stochastic demands. Each item is characterized by an ordering cost that is paid every time the item is purchased and is independent of the other products. There exists a major ordering cost that is independent of the number of items acquired and is charged with frequency established by a basic cycle time. The ordering cycle time of each product is an integer multiple of the basic cycle time. It is not required that the review period of at least one item coincides with the basic cycle time; that is, we take into account cyclic replenishment policies. Items feature a deterministic lead time made of several components that can be shortened by paying a crashing cost. In this setting, the problem consists in determining

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Table 1
Comparison among some recent works.

Refs.	Demand		Lead time		Review policy		*Backorders-lost sales mixtures*	*Cyclic replenishment policy*[a]	*Stockout costs*	Additional features
	Stochastic	*Deterministic*	*Controllable*	*Fixed*	*Periodic*	*Continuous*				
[10]	♦			♦		♦				• Capacity constraint • Service level constraint
[18]		♦		♦	♦					• Credit period • Weight freight cost discounts
[1]		♦		♦	♦			♦		
[19]		♦		♦	♦			♦		• Resource constraints • Trade credits
[20]		♦		♦	♦			♦		• Fuzzy costs • Deliveries schedule
[14]		♦		♦	♦					• Imperfect items • Price discounts
[12]		♦		♦						• Rolling horizon • Dynamic lot-size
[17]	♦			♦		♦			♦	• Capacity constraint
[15]	♦			♦	♦			♦	♦	• Integration with location decision
[23]		♦		♦	♦			♦		• Warehousing costs • Capacity constraints
[6]		♦		♦						• Dynamic lot-size
[5]		♦		♦						• Dynamic lot-size • Storage capacity constraint • Two-level inventory system
[4]		♦		♦						• Finite time horizon • Demands with deadline

[a] Only under periodic review.

the cyclic replenishment policy and the length of lead times that minimize the expected total cost per time unit.

In the next section, we will formalize both the model and the problem using the following notation:

Decision variables:

T	Basic cycle time (time units).
L_n	Length of lead time of item n (time units).
z_n	Safety factor of item n.
k_n	Integer multiplier of item n.

Parameters:

A	Major ordering cost (money/order).
a_n	Ordering cost of item n (money/order).
h_n	Unit holding cost rate of item n (money/quantity unit/time unit).
ρ_n	Fixed penalty cost per unit shortage of item n (money/quantity nit).
π_n	Marginal profit per unit of item n (money/quantity unit).
β_n	Fraction of shortage of item n that is lost.
σ_n	Standard deviation of the demand rate of item n (quantity unit/time unit).
D_n	Average demand rate of item n (quantity unit/time unit).
N	Number of items.

Random variables:

X_n	Demand of item n within the protection interval $k_nT + L_n$.

Functions and operators:

$f(\cdot)$	Standard normal probability density function.
$F(\cdot)$	Standard normal cumulative distribution function.
$G(\cdot)$	Standard normal loss function.
$E[\cdot]$	Mathematical expectation.
x^+	Maximum between 0 and x, i.e., $x^+ \equiv \max\{0, x\}$.
$\lVert\cdot\rVert$	Euclidean norm.

We also consider the following main assumptions:

1. The random variables $X_1, X_2, \ldots, X_n$ are mutually independent.
2. Inventory of item n is reviewed every k_nT time units. A sufficient quantity is ordered up to the target level R_n, and the order lot arrives after L_n time units. For each item, there is no more than a single order outstanding.
3. The target level of the nth item is given by $R_n = D_n(k_nT + L_n) + z_n\sigma_n\sqrt{k_nT + L_n}$, for $n = 1, 2, \ldots, N$, where $D_n(k_nT + L_n)$ is the average demand during the protection interval and $z_n\sigma_n\sqrt{k_nT + L_n}$ is the safety stock.
4. For each n, X_n is Gaussian with mean and standard deviation given by $D_n(k_nT + L_n)$ and $\sigma_n\sqrt{k_nT + L_n}$, respectively.
5. For each n, shortages are allowed and partially backordered with ratio $1 - \beta_n$. The fraction of shortage with ratio β_n is lost.

Similarly to, e.g., [2], we assume that the lead time L_n of item n is deterministic and made of M_n mutually independent components. The generic mth component has a minimum duration $b_{m,n}$, a normal duration $s_{m,n}$, and a crashing cost per time unit $c_{m,n}$, with $c_{1,n} \leq c_{2,n} \leq \cdots \leq c_{M_n,n}$. The components are crashed one at a time starting with the component of least $c_{m,n}$