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Quality improvement and process innovation in monopoly: A dynamic analysis

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ABSTRACT

We investigate the R&D portfolio of a monopolist investing in cost-reducing and quality enhancing R&D. Incentives along the two directions are inversely related to the size of market demand, and independent of each other. The stability analysis shows the existence of a unique stable steady state equilibrium, which is a saddle point. Finally, we show that the monopolist undersupplies product quality as compared to the social optimum, while its investment in the abatement of marginal cost is socially efficient.

1. Introduction

The impact of monopoly power on product quality is a *vexata quaestio* in the theory of industrial organisation, at least since [21] and [24], where the main issue under investigation is the firm's tendency to distort quality downwards to extract as much surplus as possible from consumers' pockets. This aspect has been largely debated (see [1,3,10,15,18,20]). This literature, however, (i) is based on static models, and therefore by construction falls short of characterising the inherently dynamic nature of quality improvement; (ii) leaves out of the picture any other form of investments, such as any effort directed at decreasing marginal production costs, and finally (iii) disregards advertising activities (either persuasive or informative) aimed at increasing demand or making the product more appealing to consumers and thus convince them to pay higher prices for it.

Here, we propose a model addressing aspects (i) and (ii), leaving aside (iii), which has generated a lively debate since the late 1970s, stemming from the pioneering contribution in [11]. The bulk of the resulting literature is summarised in [8]; for a later contribution in this vein, see [4]. We characterise the optimal R&D portfolio of a monopolist investing in cost-reducing and quality improving activities under full market coverage, and selling its product to a population of consumers endowed with different levels of willingness to pay for quality. Our results can be summarised as follows.

First, observing the control equations describing the evolution of the two R&D efforts over time, it can be established that a larger demand size exerts a negative effect on both innovation rates at any time. Second, we find that, at any time, the two innovation efforts are independent of each other, due to the assumption of full market coverage. This is in striking contrast to the extant literature on R&D portfolios, where either complementarity or substitutability between product innovation and cost reduction usually arises. The simultaneous presence of product and process innovations and their relation to product life cycle in monopoly and oligopoly models is in [16,17], using the representative consumer approach as in [23], which generates a price-elastic market demand. In these models, product and process R&D efforts may be either complements or substitutes and their relative intensity depends on initial conditions and demand parameters. Third, we prove that there exists a multiplicity of steady state points, among which a unique stable equilibrium can be singled out, this being a saddle point solution. The stability analysis is carried on a Jacobian matrix which is a block diagonal one, the latter property being due to the aforementioned fact that the two dimensions of innovation are independent of each other. Fourth, the welfare assessment reveals that the profit-maximising monopolist distorts quality downwards as compared to the social optimum, while producing the socially efficient effort along the process innovation dimension.

The remainder of the paper is structured as follows. The setup is Section 2. The equilibrium analysis is in Section 3, while Section 4

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contains the stability analysis. Section 5 examines the welfare implications.

2. The model

Our model is a variation on the setup introduced by [9] and [21]. We assume the market is supplied by a single-product monopoly selling a nondurable good of quality $q(t) > 0$ at price $p(t) > 0$ over continuous time $t \in [0, \infty)$. The population of consumers is characterised by a level of marginal willingness to pay for quality $\theta \in [\Theta - 1, \Theta]$, where $\Theta > 1$, and is distributed with a uniform density d over such interval. Hence, the total mass of consumers amounts to $d \geq 1$. Parameter θ can be interpreted as a proxy of income or wealth. A similar although not entirely equivalent and less frequent approach consists in modelling consumer preferences by describing explicitly their income distribution (see [22], *inter alia*). At any time $t \in [0, \infty)$, full market coverage is assumed. Full market coverage may be interpreted as describing a situation in which demand size is known *a priori* because under full information the firm may identify at the outset the position $\Theta - \alpha$ of the marginal consumer. Here, for simplicity and through an appropriate choice of measure, $\alpha = 1$. Each individual buys a single unit of the good, whereby his net surplus is

$$U = \theta q(t) - p(t) \geq 0. \tag{1}$$

Production takes place at marginal cost $c(t)$, which can be decreased (generating thus what is usually defined as process innovation) via an R&D effort $y(t)$. The monopolist also invests in product innovation (or quality improvement) via the effort $k(t)$, to increase $q(t)$. We assume the entire R&D activity is carried out in house by the integrated firm. For an assessment of the bearings of outsourcing on quality improvement, and the related contractual design, see [6] and [7]. The total cost function borne by the firm is

$$C(t) = c(t)x(t) + bk^2(t) + sy^2(t) + vq^2(t) \tag{2}$$

where $x(t)$ is output, while b, s and v are positive parameters. The term $vq^2(t)$ in (2) measures the instantaneous cost of producing a quality level $q(t)$ using machinery and/or skilled labour operating at decreasing returns. The state dynamics describing the evolution of $c(t)$ and $q(t)$ over time are

$$\frac{dq(t)}{dt} \equiv \dot{q} = [k(t) - \delta]\, q(t) \tag{3}$$

$$\frac{dc(t)}{dt} \equiv \dot{c} = -[y(t) - \eta]\, c(t) \tag{4}$$

in which $\delta > 0$ is the decay rate of quality while $\eta > 0$ is the obsolescence rate affecting production technology. We are supposing that R&D has an immediate impact, which is admittedly a simplifying and unrealistic assumption which, however, is commonly adopted. The presence of a decay rate in both state equations can be interpreted as the effect of technological obsolescence prevailing over learning-by-doing along both dimensions, although these two elements are not endogenously modelled. An alternative interpretation consists in thinking of the system of state dynamics (3)–(4) as perceived from the standpoint of consumers: if the firm were not investing in R&D in either direction or both, a consumer could think of the product as one which incorporates an old and therefore inferior technology or know-how.

Under full market coverage, $x(t) = d$ and the profit-maximising price extracts the entire surplus from the pockets of the poorest consumer, i.e., it is $p^m(t) = (\Theta - 1)\, q(t)$, with superscript m standing for *monopoly* (cf. [12, p. 113]). The monopolist's instantaneous profits are

$$\pi(t) = [(\Theta - 1)\, q(t) - c(t)]\, d - bk^2(t) - sy^2(t) - vq^2(t) \tag{5}$$

and the firm wants to maximise the discounted profit flow

$$\Pi(t) = \int_0^\infty \pi(t)\, e^{-\rho t} dt \tag{6}$$

w.r.t. controls $k(t)$ and $y(t)$, under the constraints posed by the state equations (3)–(4), initial conditions $q(0) = q_0 > 0$, $c(0) = c_0 \in (0, (\Theta - 1)\, q(0))$, and the appropriate transversality conditions to be specified below. It is worth observing that the initial condition on marginal cost says that it must be strictly lower than the spending capability of the poorest consumer existing in this market, in order for full market coverage to hold at $t = 0$. Profits are discounted at the constant rate $\rho > 0$.

3. Equilibrium analysis

The firm's current value Hamiltonian is

$$\mathcal{H} = e^{-\rho t}\left(\pi + \lambda \dot{q} + \mu \dot{c}\right) \tag{7}$$

where $\lambda = \zeta e^{\rho t}$ and $\mu = \psi e^{\rho t}$ are the costate variables (evaluated at time t) associated with q and c, respectively. Henceforth, we shall omit the explicit indication of the time argument for the sake of brevity. The resulting first order conditions (FOCs) on controls and costate equations are (exponential discounting is omitted for brevity):

$$\frac{\partial \mathcal{H}}{\partial k} = -2bk + \lambda q = 0 \tag{8}$$

$$\frac{\partial \mathcal{H}}{\partial y} = -2sy - \mu c = 0 \tag{9}$$

$$\dot{\lambda} = -\frac{\partial \mathcal{H}}{\partial q} + \rho\lambda \Leftrightarrow \tag{10}$$

$$\dot{\lambda} = (\delta + \rho - k)\,\lambda - d(\Theta - 1) + 2hq$$

$$\dot{\mu} = -\frac{\partial \mathcal{H}}{\partial c} + \rho\mu \Leftrightarrow \tag{11}$$

$$\dot{\mu} = (\rho - \eta + y)\,\mu + d.$$

The accompanying set of transversality conditions is $\lim_{t\to\infty} \lambda q e^{-\rho t} = 0$ and $\lim_{t\to\infty} \mu c e^{-\rho t} = 0$.

From (8)–(9), we have the optimal controls at time t:

$$k^* = \max\left\{0, \frac{\lambda q}{2b}\right\}; \qquad y^* = \max\left\{0, -\frac{\mu c}{2s}\right\} \tag{12}$$

and the control equations

$$\dot{k} = \frac{\dot{\lambda} q + \lambda \dot{q}}{2b}; \qquad \dot{y} = -\frac{\dot{\mu} c + \mu \dot{c}}{2s} \tag{13}$$

which, using (8)–(9) and (12), can be rewritten as follows:

$$\dot{k} = \rho k - \frac{q[(\Theta - 1)\, d - 2vq]}{2b} \tag{14}$$

$$\dot{y} = \rho y - \frac{cd}{2s}. \tag{15}$$

The system composed by (3)–(4) and (14)–(15) identifies the state-control system of the dynamic problem at hand. In particular, the above control equations show that the instantaneous R&D rates in both directions is decreasing in the density parameter d. Given that d in this model also measures the total mass of consumers, we may formulate:

Lemma 1. *The instantaneous investment rates in product and process innovation are inversely related to the size of demand.*