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Pre-announced posted pricing scheme: Existence and uniqueness of equilibrium bidding strategy

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ABSTRACT

We scrutinize the uniqueness issue of the equilibrium behavior of strategic customers making inter-temporal purchase decisions. We present that multiple equilibria can exist even in the simple setting where two identical customers compete for one unit of item. We prove the existence of a unique symmetric equilibrium when there are no more than three buyers whose valuations follow a uniform distribution. When there are multiple equilibria we show that the bidding strategy having the biggest threshold is Pareto-optimal for buyers.

1. Introduction

We study the equilibrium behavior of strategic customers who make inter-temporal purchasing decisions in the presence of a markdown pricing scheme. Temporal price changes are prevalently used in practice — ranging from the simple markdown pricing in the fashion or retail industry to more complicated pricing schemes in the airline industry. At the same time, customers have also become strategic in their decision making as they factor in several issues into consideration, such as the timing of the purchase, future price, and in-stock probability. Understanding strategic customer behavior becomes a critical part for firms to design pricing schemes. See, for example, [1,7,8] for comprehensive literature reviews on dynamic pricing and strategic customer behavior.

In the posted pricing scheme, a path of prices is announced to the customers at the beginning of the planning horizon. Since the posted pricing scheme eliminates the uncertainty of the future prices, it simplifies the decision making of strategic customers and, therefore, offers a simpler theoretical ground for more sophisticated pricing schemes. This pricing scheme is studied by a number of researchers such as [2,4,5]. They investigate the best response for each customer, find the Nash equilibrium of the customers' purchasing strategies, and based on this equilibrium, they have extended the analysis to the impact of this strategic behavior on the seller's decision and the expected revenue.

In this paper, we focus on the uniqueness issue of the buyers' equilibrium bidding behavior. We consider a setting where a number of customers compete for items offered by a monopolist in two periods. The number of items is always less than the number of customers. The customers are identical in the sense that their valuations are drawn from the same distribution. Similar to [4], the customers arrive at the start of selling season. Prices are preannounced in the beginning, and the second-period price represents markdown. Even in the simple setting of two customers and one item, we find some examples showing multiple equilibria.

While the unique equilibrium behavior is often essential to meaningfully predict the outcomes of a game with the strategic customers, this issue has received limited attention in the literature. For example, [4] does not acknowledge that the buyer's equilibrium bidding strategy may not be unique. [3,6] have investigated uniqueness in their extensions to the two-period posted pricing scheme of [2]. [6] offers a sufficient condition for the uniqueness of an equilibrium, and [3] shows the existence of unique equilibrium in a more general setting with one item.

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These papers assume that the buyers arrive dynamically according to a Poisson process, and establish the uniqueness of the equilibrium. They have considered a setting in which the buyers bid sequentially (one at a time). This is different from our assumption that the buyers bid simultaneously. Furthermore, [3,6] show that the buyer's equilibrium bidding strategy may not be unique.

In the case of multiple equilibria, they show that the threshold policy having higher in-stock probability is Pareto-optimal. In our paper, we consider a setting where all of the buyers present from the beginning of the planning horizon, and show that multiple equilibria can exist even if the firm offers only one unit of the item. The model by [5] adopts an assumption that "valuations are drawn from nonoverlapping intervals", which sidesteps the difficulty associated with proving uniqueness by assuming that only one of the two buyers faces a nontrivial bidding decision.

2. Model description

Our model is adapted from [4,5]. Suppose that there exist $\overline{N}$ buyers, indexed by $i = 1, \ldots, \overline{N}$, and each buyer is interested in obtaining one unit of a particular product. Let V^i denote the random variable representing buyer i's valuation of the product, and we use v^i to denote its realization. We assume that $\{V^1, \ldots, V^{\overline{N}}\}$ are independent and identically distributed with the common distribution V, and we refer to its cumulative density function and probability density function as $G(\cdot)$ and $g(\cdot)$, respectively. Let K be the number of units for sale. The price changes from the first period to the second period, and let p_1 and p_2 denote the price per unit in the first period and the second period, respectively. Both p_1 and p_2 are exogenously given, and we assume $p_1 \geq p_2$.

We describe the sequence of events. The value of v^i is realized for each i, and any customer i with valuation $v^i \leq p_2$ leaves the system immediately. Then, the remaining buyers decide, independently and simultaneously, whether or not to place a bid in the first period. If the seller (monopolist) receives K or more bids, then she randomly selects, with equal probability, the K buyers to whom the unit will be sold at the price of p_1, and there is no more unit remaining for the second period. If the number of bids is less than K, then each buyer who bids will buy the product at p_1, and any remaining unit will be sold in the second period at p_2, when buyers who did not buy in the first period have an opportunity to bid.

This model offers a simple structure to analyze the buyer's problem in the second period. If he does not place a bid, then his payoff will be 0. If he places a bid, then there is some chance being able to buy a unit, in which case his payoff is $v - p_2$, where v is the buyer's valuation of the product. Thus, it is optimal that the buyer places his bid if and only if his valuation exceeds p_2. The first period problem involves a more delicate tradeoff between the price and the probability of obtaining the product. Below, we characterize the buyers' equilibrium bidding strategies and show that they follow a threshold policy.

Lemma 1. *Fix p_1 and p_2, where $p_1 \geq p_2$. Let $i \in \{1, \ldots, \overline{N}\}$.*

(a) *The dominant strategy of buyer in the second period is to bid if and only if $v^i \geq p_2$.*
(b) *Any equilibrium among the buyers can be characterized by $\tau^i \geq p_1$, for each $i \in \{1, \ldots, \overline{N}\}$ such that buyer i's strategy is given by*

$$\begin{cases} \text{do not bid} & \text{if } v^i < p_2 \\ \text{bid in period 2} & \text{if } p_2 \leq v^i < \tau^i \\ \text{bid in period 1} & \text{if } v^i \geq \tau^i. \end{cases} \tag{1}$$

Proof. If the second period bids are accepted, then the buyer's expected profit from bidding in the second period is $(v^i - p_2)$ multiplied by the probability that the buyer will obtain a unit in the second period conditioned on bids being accepted in the second period. Since this depends only on the other buyers' strategies, not on v^i, the optimal decision for buyer i is to bid if and only if $v^i \geq p_2$. This proves (a).

Now, it suffices to consider the first period bid for the case $v^i \geq p_2$. Suppose that we fix the strategies of buyers other than buyer i. Let π_1 and π_2 denote the probability that buyer i will obtain a unit if he bids in the first period and if he bids in the second period, respectively. Clearly $\pi_1 \geq \pi_2$. Suppose that it is optimal to bid in the first period with v^i. Then,

$$\pi_1 \cdot (v^i - p_1) \geq \pi_2 \cdot (v^i - p_2).$$

Then, for any $\hat{v} > v^i$, we have

$$\begin{aligned} 0 &\leq \pi_1 \cdot (v^i - p_1) - \pi_2 \cdot (v^i - p_2) \\ &\leq \pi_1 \cdot (v^i - p_1) - \pi_2 \cdot (v^i - p_2) + (\pi_1 - \pi_2) \cdot (\hat{v} - v^i) \\ &= \pi_1 \cdot (\hat{v} - p_1) - \pi_2 \cdot (\hat{v} - p_2)\,, \end{aligned}$$

where the second inequality follows from $\pi_1 \geq \pi_2$ and $\hat{v} > v^i$. Thus, we obtain $\pi_1 \cdot (\hat{v} - p_1) \geq \pi_2 \cdot (\hat{v} - p_2)$, implying that it is also optimal for buyer i to bid in the first period when his value is $\hat{v}$. □

We remark that Lemma 1 is applicable for a general model with arbitrary $\overline{N}$, K and any distribution for V.

3. Multiplicity and uniqueness of the buyers' equilibrium strategy

3.1. Multiplicity of equilibrium

We first show that the equilibrium strategy for buyers may not be unique.

Proposition 2. *Suppose $\overline{N} = 2$ and $K = 1$. Suppose that the buyers' valuation is deterministic and identical, i.e., $V^1 = V^2 = \hat{v}$. If $p_1 \leq \hat{v} \leq 2p_1 - p_2$, then the buyers' bidding strategy forms multiple equilibria.*