Operations Research Letters 41 (2013) 172–179

Contents lists available at SciVerse ScienceDirect

Operations Research Letters

journal homepage: www.elsevier.com/locate/orl

The carbon-constrained EOQ

Xi Chen [a], Saif Benjaafar [a,b,*], Adel Elomri [a]

[a] Department of Industrial and Systems Engineering, University of Minnesota, Minneapolis, MN 55455, USA

[b] Engineering Systems and Design, Singapore University of Technology and Design, Singapore 138682, Singapore

ARTICLE INFO

Article history:
Received 1 August 2011
Accepted 12 December 2012
Available online 26 December 2012

Keywords:
Inventory management
Economic order quantity
Carbon emissions
Sustainable operations
Environmental regulations

ABSTRACT

In this paper, we provide analytical support for the notion that it may be possible, via operational adjustments alone, to significantly reduce emissions without significantly increasing cost. Using the EOQ model, we provide a condition under which it is possible to reduce emissions by modifying order quantities. We also provide conditions under which the relative reduction in emissions is greater than the relative increase in cost and discuss factors that affect the difference in the magnitude of emission reduction and cost increase. We discuss the applicability of the results to systems under a variety of environmental regulations, including strict carbon caps, carbon tax, cap-and-offset, and cap-and-price.

1. Introduction

In pursuing carbon emission reduction efforts, firms have focused for the most part on reducing emissions due to the physical processes involved (e.g., replacing energy inefficient equipment and facilities, redesigning products and packaging, deployment and use of less polluting sources of energy); see [3], the unabridged version of the paper. These efforts are clearly valuable. However, they can overlook a potentially significant source of emissions, one that is driven by business practices and operational policies. In this paper, we examine the extent to which operational adjustments alone could indeed be effective in reducing emissions. We also examine the extent to which such adjustments could take place without significantly increasing cost. This is important because resistance to environmental regulation has often been based on concerns that such regulation would lead to significantly higher costs.

Our analysis is in part motivated by a recent paper [1], in which the authors observe that it is possible to significantly reduce carbon emissions without significantly increasing cost by making only operational adjustments. Their observations are based on numerical results obtained for a lot sizing problem in which a firm decides on production/procurement quantities over a finite planning horizon consisting of discrete periods. In this paper, we use the framework of the economic order quantity (EOQ) model to provide analytical support for similar observations. We provide a condition under which it is possible to reduce emissions by modifying order quantities. We also provide conditions under which the relative reduction in emissions is greater than the relative increase in cost and describe when the difference between the two is maximized. We discuss the applicability of these results to systems operating under a variety of regulatory policies and to other operational models. We show that, the key requirements are that the cost and emission functions yield different optimal solutions, implying that the cost tradeoffs are different from the emission tradeoffs, and that the cost function is flat around the optimal solution but can be steep elsewhere. We show that significant reductions in emissions can indeed be achieved without significant increases in cost whenever the flat region of the cost function coincides with the steep region of the emission function. In the unabridged version of the paper [3], we show that these features are present in other operational models, including the facility location and newsvendor models, among others.

The results in this paper indicate that the opportunity for reducing carbon emissions via operational adjustments exists whenever the operational drivers of emissions are different from the operational drivers of costs. In settings where this is not the case (e.g., operational decisions that reduce cost tend to also reduce emissions), operational adjustments will obviously be ineffective. In that case, investments in efforts that modify the emission function (e.g., investments in efforts or technologies that lead to reductions in the emission parameters of underlying processes and activities) would be necessary.

Although there is growing literature that is concerned with issues of sustainability in operations (see [1] for a review), papers that explicitly consider emissions are relatively few. Hua et al. [6] consider a model similar to the cap-and-price model

* Corresponding author at: Department of Industrial and Systems Engineering, University of Minnesota, Minneapolis, MN 55455, USA.

E-mail addresses: chen1644@umn.edu (X. Chen), saif@umn.edu (S. Benjaafar), aelomri@umn.edu (A. Elomri).

0167-6377/$ – see front matter
doi:10.1016/j.orl.2012.12.003

we consider in Section 3. They compare the cost and order quantity obtained under cap-and-price to those obtained without carbon considerations. Cachon [2] studies the emission tradeoffs associated with the size and location of retail facilities and shows that carbon pricing would have little impact on these decisions; see also [5,8] for related analysis.

2. Problem formulation and results

Consider a firm that faces a constant demand with rate D per unit time. Each time the firm places an order (either with its internal production facility or with an external supplier), it incurs a fixed cost A per order. The firm also incurs a holding cost h per unit kept in inventory per unit time, and a cost c per unit purchased or produced. Without loss of generality, we assume that orders are delivered with zero lead time (a positive lead time can be included and does not affect the solution to the problem); we also assume that the firm must satisfy all the demand (the analysis can be easily extended to settings with backorders). Total cost per unit time is then given by

$$\frac{AD}{Q} + \frac{hQ}{2} + cD.$$

Similar to cost, emissions are associated with ordering, inventory holding, and production/purchasing, with $\hat{A}$, $\hat{h}$ and $\hat{c}$ denoting the amount of carbon emissions associated per order initiated, per unit held in inventory per unit time, and per unit purchased or produced. Total emission per unit is therefore

$$\frac{\hat{A}D}{Q} + \frac{\hat{h}Q}{2} + \hat{c}D.$$

The parameterization of emissions under the above formulation is flexible and can be used to capture a variety of settings. For example, if emission from holding inventory depends only on the maximum amount of inventory held (which would correspond to Q), then total emission can be expressed as $\frac{\hat{A}D}{Q} + \hat{h}Q + \hat{c}D$, which is equivalent to the original expression but with an inventory holding emission parameter equal to $2\hat{h}$. Similarly, if emission from holding inventory is invariant to the amount of inventory held, then total emission reduces to $\frac{\hat{A}D}{Q} + \hat{h} + \hat{c}D$, which is equivalent to the original model but with an inventory holding emission parameter equal to 0. If the emission associated with initiating an order has both a fixed and a variable component, say of the form $\hat{A}_1 + \hat{A}_2 Q$, then the corresponding total emission is $\frac{\hat{A}_1 D}{Q} + \frac{\hat{h}Q}{2} + (\hat{c} + \hat{A}_2)D$, which again has the same form as the original model. Note that, depending on the setting, $\hat{A}$ can be higher or lower than $\hat{h}$. For example, for some products transportation-related emissions are high but storage emissions are low or even negligible (e.g., canned foods) while for others the reverse may be true (e.g., refrigerated foods). Walmart recently discovered that the refrigerants used in grocery stores accounted for a larger percentage of Walmart's greenhouse gas footprint than its truck fleet (http://www.walmartstores.com/sites/responsibility-report/2012/sustainableFacilities.aspx). Tesco, the largest retailer in the UK, found that 26 percent of its direct emissions were due to refrigerant leakage while only 12 percent were due to transportation (http://www.tesco.com/climatechange/carbonFootprint.asp). Our analysis and results are applicable in all cases (see the end of this section for additional discussion).

The objective of the firm is to choose an order quantity Q that minimizes its cost per unit time subject to the constraint on the amount of carbon emitted (this cap can reflect either government regulations imposed on the firm or a voluntary effort by the firm to reduce its emissions by a specified amount). The amount of carbon emitted is constrained to be less than a certain cap C. The problem can then be formally stated as follows:

$$\text{Minimize } Z(Q) = \frac{AD}{Q} + \frac{hQ}{2} + cD \quad (1)$$

$$\text{subject to } \frac{\hat{A}D}{Q} + \frac{\hat{h}Q}{2} + \hat{c}D \leq C. \quad (2)$$

Let $\hat{Q}_{\min}$ denote the order quantity that minimizes carbon emission (the *emission-optimal* solution), then it is easy to verify that $\hat{Q}_{\min} = \sqrt{\frac{2\hat{A}D}{\hat{h}}}$ and the corresponding emission level is $E_{\min} = \sqrt{2\hat{A}\hat{h}D} + \hat{c}D$. Consequently, the problem admits a feasible solution if and only if $C \geq E_{\min}$. In the remainder, we assume that this condition is always satisfied. Also, let Q^* denote the order quantity that minimizes the total cost while ignoring the carbon emission constraint (the *cost-optimal* solution). Then, it is easy to see that $Q^* = \sqrt{\frac{2AD}{h}}$, which corresponds to the standard EOQ solution. The following theorem characterizes the optimal solution to (1)–(2).

Theorem 1. *Let*

$$Q_1 = \frac{\hat{C} - \sqrt{\hat{C}^2 - 2\hat{A}\hat{h}D}}{\hat{h}} \quad \text{and} \quad Q_2 = \frac{\hat{C} + \sqrt{\hat{C}^2 - 2\hat{A}\hat{h}D}}{\hat{h}}$$

where $\hat{C} = C - \hat{c}D$. Then the optimal solution to problem (1)–(2) is

$$\hat{Q}^* = \begin{cases} Q^* & \text{if } Q_1 \leq Q^* \leq Q_2, \\ Q_1 & \text{if } Q^* \leq Q_1, \\ Q_2 & \text{if } Q^* \geq Q_2. \end{cases}$$

Furthermore, the emission level under the optimal order quantity is

$$E(\hat{Q}^*) = \begin{cases} E_{\max} & \text{if } Q_1 \leq Q^* \leq Q_2 \\ C & \text{otherwise,} \end{cases}$$

where

$$E_{\max} = \hat{A}\sqrt{\frac{hD}{2A}} + \hat{h}\sqrt{\frac{AD}{2h}} + \hat{c}D$$

and corresponds to the emission level in the absence of the carbon constraint (also corresponds to the emission level when the optimal order quantity is Q^*).

Proof. From constraint (2), we can show that the optimal order quantity must satisfy $Q_1 \leq \hat{Q}^* \leq Q_2$. If $Q_1 \leq Q^* \leq Q_2$, then obviously $\hat{Q}^* = Q^*$ and the corresponding emission is

$$E(Q^*) = \frac{\hat{A}D}{Q^*} + \frac{\hat{h}Q^*}{2} + \hat{c}D = \hat{A}\sqrt{\frac{hD}{2A}} + \hat{h}\sqrt{\frac{AD}{2h}} + \hat{c}D.$$

If $Q^* \leq Q_1$ then $\hat{Q}^* = Q_1$ because $Z(Q)$ is convex in Q and choosing a higher value for $\hat{Q}^*$ will lead to a higher cost. Similarly, if $Q^* \geq Q_2$ then $\hat{Q}^* = Q_2$ because choosing a lower value for $\hat{Q}^*$ will lead to higher cost. In both of these cases, constraint (2) is binding and, therefore, $E(\hat{Q}^*) = C$. □

In the following proposition, we show that cost is indeed decreasing and convex in the emission cap C while emission is linearly increasing in C, implying that reducing the emission cap leads initially to a larger relative emission reduction than the relative cost increase (e.g., in the example illustrated in Fig. 1, an emission reduction of 20% leads only to a 4% increase in cost).

Proposition 1. *For $C \geq E_{\min}$, emission is linearly increasing in C while cost is decreasing and convex in C.*