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On the replenishment policy considering less expensive but non-committed supply

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ABSTRACT

We study an inventory system that replenishes its stock through two shipping services, of which the non-committed shipping service can only ship up to a random capacity that is not known until shipment. By transforming the non-convex optimization in the dynamic program to a convex optimization problem, we show that the optimal policy for each period is determined by a quota for the non-committed shipping and a base-stock level for the committed shipping.

1. Introduction

This study is motivated by a retailer who replenishes its stock through an air carrier that implements a dual-pricing mechanism. The air carrier, headquartered in the southern city of Guangzhou, China, recently instituted the "committed" versus "non-committed" air shipping products for its customers. For convenience in what follows we shall refer to these two shipping services as *committed and non-committed shippings*, respectively; and refer to the respective orders as *committed and non-committed orders*. The committed order is always guaranteed to ship when received, and it is charged a higher shipping cost; the non-committed order has a lower shipping cost, but the capacity to ship non-committed orders is not known until all committed orders have been received. That is, the capacity for non-committed orders is random and that is not known to the retailer when placing its order; thus a non-committed order is not guaranteed of the delivery of the exact ordering quantity. (These air cargo shipping products have been proven to be very successful in increasing the carrier's revenue. For more information on these air cargo products, see Wu [11]). This raises the following interesting operational question for a retailer whose inbound logistics are handled by such a shipping carrier: How to take advantage of the less-expensive but non-committed shipping to minimize its total cost?

The main complication of the problem lies in the random and unknown capacity of the non-committed shipment when the retailer makes its replenishment decisions. Indeed, even for the special case of only one (the non-committed) shipping service, it is known that the random capacity results in an optimization problem that is not convex (see e.g., Ciarallo et al. [2], and Feng and Shi [5]). One of the contributions of this paper is to transform the non-convex optimization problem into a simple convex optimization problem. This transformation allows us to easily derive the optimal strategy for the retailer. We show that the optimal control policy for each period is determined by a *quota* allocated to the non-committed shipping and a *base-stock level* for the committed shipping. We also study the effect of introducing the non-committed shipping as well as the effect of a price change of the non-committed shipping to the retailer. We prove that adding the option of non-committed shipping always benefits the retailer, and it leads to a reduced optimal base-stock level for the committed order in each period. For ease of exposition we assume that the data is stationary, but all results hold true when the system costs, demand distributions, etc., are time-dependent.

2. Related literature

This work is closely related to two streams of research literature. As the ordering cost for the committed shipping becomes exceedingly high, then the problem is reduced to an inventory system

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with only one supplier that has random capacity, which is the problem studied in Ciarallo et al. [2], and more recently, in Feng [4]. As mentioned above, since the unknown random capacity leads to a non-convex optimization problem, the analysis of the non-convex objective function is the key to these studies. Ciarallo et al. [2] employ an interesting quasi-convexity analysis to show that the value function (cost-to-go function) is convex and the optimal policy is a base-stock type. The resulting optimization problem in Feng and Shi [5] is also non-convex in the ordering decisions, and the authors use a novel approach to show that the value function is convex (in the profit maximization setting it is concave) and that there exists a threshold for each supplier and the retailer would order only if the starting inventory level is below the threshold, but the ordering quantity can be complicated. In contrast to Ciarallo et al. [2] and Feng and Shi [5], our approach transforms the non-convex optimization problem to a convex optimization problem, and the transformed convex optimization problem significantly simplifies the analysis that establishes the optimality of quota-base-stock optimal policy and the effect of introducing the non-committed shipping. The policy of assigning a quota to the non-committed shipping offers managerial insight; and so is the effect of introducing the non-committed shipping that lowers the retailer's minimum cost and reduces the optimal base-stock levels.

On another extreme, if the capacity of the non-committed shipping service is deterministic, then the problem is reduced to a periodic-review inventory system with only one supplier and piece-wise linear convex ordering cost: the unit ordering cost is a low number up to the fixed capacity, and a high number beyond the capacity. This problem has also been studied by a number of authors, and the optimal inventory control policy is characterized by two critical numbers together with the deterministic capacity, see e.g., Henig et al. [7], Bensoussan et al. [1], and Martinez-de-Albeniz and Simchi-Levi [9], among others. The piece-wise cost structure is due to capacity constraints and the model in this paper further allows the capacity to be unknown at the decision time. Our model is also related to the research in which the firm needs to dynamically make ordering decisions from a regular supplier and a spot market to minimize the total cost, see, e.g., Yi and Scheller-Wolf [12] and the survey paper of Haksoz and Seshadri [6]. The difference with this line of work is that, in the spot market there are usually plenty of supplies while in our model, there is a finite capacity that is not known beforehand.

3. Notations and problem setup

Consider a periodic-review inventory system of a retailer for N periods. The first period is labeled 1, and the last period is N. The random demands in these periods $D_1, D_2, \ldots, D_N$ are independent though not necessarily identically distributed. In this paper we consider the case that unmet demand in each period is backlogged. There is a discount factor α for each period for cost computation, where $\alpha \in (0, 1]$.

Two types of shipping services, committed shipping and non-committed shipping are available to the retailer. Committed shipping promises to deliver any orders received in a period. Due to the commitment, the carrier has taken necessary actions to secure the capacity for these orders, including scheduling in advance, emergency capacity allocation, rejecting other potential customers, and so on, thus the retailer incurs a high unit ordering cost c^H for committed shipping. On the other hand, non-committed shipping does not guarantee to deliver the full ordering quantity. More specifically, the delivery quantity of the non-committed shipping depends on the realized remaining capacity in the period, say R_n, for period n. This is due to the fact that the carrier has priority customers that have to be satisfied first, and only when extra capacity is available after serving the priority customers shall the non-committed orders be delivered, and for this reason, the non-committed orders are charged at a lower price $c^L \leq c^H$. We assume that the capacities $R_1, \ldots, R_N$ are independent, which is a reasonable assumption when the carrier has a large customer base. Thus the remaining capacity for non-committed orders does not depend on a small number of customers. The ordering delivery leadtimes from both shipping service are equal and are assumed, without loss of generality, to be 0. Thus, if the non-committed shipping service does not ship the full ordering quantity, we assume that the unmet order is canceled since the firm can adjust its order quantities in the following period.

Let $G(z)$ be the expected holding and shortage cost in a period if the realized inventory level after ordering decisions is z. We assume that $G(z)$ is convex in z. A typical example for $G(\cdot)$ is

$$G(z) = hE[(z - D_n)^+] + bE[(D_n - z)^+],$$

where h and b are, respectively, the holding and shortage cost rates. The goal of the retailer is to find the optimal ordering policy from the committed and non-committed shipping services so that its expected total discounted cost over the planning horizon of N periods is minimized.

Suppose at the beginning of period n, x is the starting inventory level, y is the inventory level after committed order is placed, and Q is the non-committed ordering quantity. Then, the delivery quantity from the committed shipping is $y - x$, and the delivery quantity from the non-committed shipping is $\min\{Q, R_n\}$. Therefore, the inventory level after the committed order and non-committed order is $y + \min\{Q, R_n\}$. Recall that $G(z)$ is the expected holding and shortage cost in period n if the realized inventory level after ordering decisions is z. Hence, the expected holding and shortage cost in period n after ordering decisions, y and Q, is $E_{R_n}[G(y+\min\{Q, R_n\})]$.

Let $V_n(x)$ be the minimum expected total discounted cost from period n until the end of the planning horizon. Given that the starting inventory level is x, then it satisfies the following Bellman equation:

$$\begin{aligned} V_n(x) = \min_{y \geq x, Q \geq 0} \{ & c^H(y - x) + c^L E_{R_n}[\min\{Q, R_n\}] \\ & + E_{R_n}[G(y + \min\{Q, R_n\})] \\ & + \alpha E_{R_n, D_n}[V_{n+1}(y + \min\{Q, R_n\} - D_n)]\}. \end{aligned} \tag{1}$$

As usual, we let the terminal condition be $V_{N+1}(\cdot) \equiv 0$.

Ciarallo et al. [2] study a special case of the problem above with only non-committed orders. In that work, the authors use quasi-convexity analysis to first show that the objective function is convex before the minimum point, and is increasing beyond the minimum point. Then, they continue to show that the value function is actually convex. In this paper, we solve the more general optimization problem above using a much simpler approach.

4. Results and analysis

The following simple observation allows us to significantly simplify the analysis of the optimal control policy. Indeed, this result enables us to avoid the quasi-convexity analysis. Basically, this result states the following obvious fact: if a solution optimizes a random system for every sample path, then it also optimizes the system for the average value.

Lemma 1. *Let $g(x, R)$ be a function of x and random variable R, thus its value depends on the realization of R. Suppose that for any realization of R, $g(x, R)$ is minimized at $x = x^*$ that is independent of R, then x^* is also a minimizer of $E_R[g(x, R)]$. In general, if the infimum of $g(x, R)$ over x is independent of random variable R, then*

$$\inf_x E_R[g(x, R)] = E_R[\inf_x g(x, R)],$$

whenever the expectations exist.