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An auction with positive externality and possible application to overtime rules in football, soccer, and chess

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ABSTRACT

We analyze auctions with positive externality, wherein the utility of each player who submitted a losing bid is strictly increasing in the price paid by the winning bidder. Such an auction was recently proposed for determining the starting team and the starting yard line in an overtime period in American football. We analyze the NFL case and also consider other football leagues, as well as tie-breaking by penalty shots in soccer, and overcoming a draw situation in chess.

1. Introduction

The fast-growing role of auctions in the economy is primarily due to their effective allocative properties (see, e.g., [8,9]). Some types of auctions inherently encourage "true bids", reflecting the bidders' true valuations, which are economically efficient. Indeed, the well-known economic advantage of a second-price auction over a first-price one is that the former induces truthful bids, while the latter does not [12]. In most common types of auctions, the bidders who do not win are not influenced by the magnitude of the winning bid. One exception is a "knockout auction" (see, e.g., [4,3]), used for the allocation of jointly owned indivisible items, where the winner pays the losers their shares of the winning bid (there is no seller). A motivation for the present work is another type of auction, recently proposed in [1] for determining the starting team and starting position in the overtime period in National Football League (NFL) games. Instead of determining the starting team by a coin toss, as is currently practiced, in the proposed auction the teams compete over who will be the starting team by submitting bids for the starting position in a manner to be explained in the sequel. We also consider bidding possibilities for starting overtime periods in NCAA and Canadian football, tie-breaking by penalty kicks in soccer, for which we provide a more formal analysis, and overcoming a draw outcome in chess competitions. We show, for example, that the auction mechanism proposed in [1] to determine the team that will be first on offense at the start of the overtime period in the NFL will not induce truthful bidding, and that first-price and second-price auction mechanisms in these situations will induce underbidding and overbidding, respectively, by the teams.

2. Bidding with externalities

In most types of auctions, the losing bidders do not incur any post-bid externalities. However, a *negative* externality, say, could occur to a losing bidder if a win in the auction by a rival player results in a decrease in his market power or market share. Consider an auction for an item with two bidders, and assume that the true valuation for the item for bidder i is v_i, $i = 1, 2$, but a loss by bidder i has a negative externality to him of value u_i, $i = 1, 2$, where $u_i \geq 0$. Then, it can be easily argued that the true valuation of bidder i is $v_i + u_i$, $i = 1, 2$, and, for example, in a second-price sealed-bids auction, the players will bid their true valuations. This observation can be generalized to many bidders. Suppose the value of an auctioned item for bidder i is v_i, his externality cost if player j, $j \neq i$, wins the item is u_j, $u_j \geq 0$, and assume the existence of a vector, P^i, providing the prior conditional probability that player j, for all $j \neq i$, will win the item, given that player i did not win the auction. Then, for example, if bidder i is risk neutral, the expected externality cost to bidder i if he does not win the item is $E_i \equiv \Sigma_{k \neq i} P^i_k u_k$, and in a second-price auction, bidder i will bid his true valuation, $v_i + E_i$.

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Next, consider the following scenario, where two players, 1 and 2, submit sealed bids, b_i, $i = 1, 2$, respectively, and the player with the highest bid (ignoring ties) wins the item and pays for it a weighted average, $\alpha b_1 + (1-\alpha)b_2$, of the two bids, for $0 \le \alpha \le 1$. Each of the players would like to own the item but we will assume that in this scenario, if a player loses, because he submitted the lower bid, he strictly prefers that the winning player pays more for the item rather than less. That is, we will consider in this note auction situations, which we will refer to as *auctions with positive externality*, in which the utility function of the losing bidder strictly increases with the price for the item paid by the winning bidder.

Note that knockout auctions can be naturally viewed as auctions with positive externality of the type discussed above. Indeed, losing bidders in a knockout auction strictly prefer the winning bid to be as large as possible, since a higher winning bid increases the payment they receive from their share of the indivisible object.

3. Application to overtime rules in football, chess, and soccer

Overtime in the National Football league (NFL)

In the NFL, a tied football game goes into a sudden death overtime period. In regular season games, the overtime period is 15 min and the game can end up in a tie. In playoff games, there must be a winner, and the game continues until someone scores. If the starting team in the overtime period scores a touchdown on its first possession, the game is over, and the scoring team is the winner. If the starting team only scores a field goal or fails to score on its first possession, the other team gets possession of the ball (after a kickoff if a field goal was scored), and has a chance to tie the game, if the first team scored a field goal, or win the game. If the first team scored a field goal on its first possession and the second team failed to tie the game on its first possession, the starting team is the winner. Otherwise, the game continues until one of the team scores (a touchdown or a field goal) or, in regular season games, until the period is over, in which case the game ends with a tie. The key question is how does the overtime period start—which team is first on offense and from where (distance to the end zone) it starts. Naturally, the team which is first on offense has a clear advantage. If it scores a touchdown, the game is over. The current rule is to determine possession by a coin toss, and have a kickoff. However, many in football seem to be uncomfortable to base such an important decision on a coin toss ([1] and media quotes therein). While a coin toss is fair ex-ante, it is not "fair" ex-post. The loser in the coin toss would like to be the winner. Indeed, according to [2], of the 24 non-tied overtime games played so far under the new overtime rules introduced by the NFL (field-goal exclusion rule), 16 (2/3) were won by the receiving team and eight (1/3) by the kicking team. In other words, the winner of the coin toss enjoyed, on average, a 2:1 edge in overtime. To overcome the difficulties stemming from a coin toss, an auction mechanism is suggested in [1] according to which each team submits a bid for the yard line from which play starts. The yard line selected will be the average of the two bids, with the low bidder playing defense and the high bidder playing offense. To illustrate, suppose Team A_1 submits a bid of 75 yard line, meaning it is prepared to start from its own 25 yard line if it is the first team to get the ball, and Team A_2 submits a bid of 70 yard line, then Team A_1 will start on offense, Team A_2 will start on defense, and the game will start 72.5 yards away from Team A_2's end zone.

A true valuation, v_i, of Team A_i in the above football example is the furthest distance from Team A_j's, $j \neq i$, end zone it is prepared to start from, if it is the first team on offense. At exactly v_i yards away from its opponent's end zone, it is indifferent whether it gets to be the first team on offense or be on defense, $100 - v_i$ yards away from its own end zone. However, note that if a team loses the bid, it strictly prefers that the other team will start further away from its own end zone, which casts the football bidding case as an auction with externality discussed above.

Overtime in American college football and Canadian football

As in the NFL, in American college football, if the teams are tied at the conclusion of regulation time, an overtime period ensues. The overtime period begins with a coin toss to determine who gets possession and who defends their goal first. Teams who win the coin toss overwhelmingly prefer to defend their goal first (see, e.g., [11,1]). Unlike overtime in the NFL, college football overtime allows each team the chance to have possession. The team that gets possession first receives the ball on its opponent's 25-yard line. The team can keep possession of the ball until it either: (i) scores a touchdown, (ii) attempts a field goal, (iii) runs out of downs, or (iv) turns over the ball. Once the first team's possession is over, the second team gets possession and follows the same format. If the first team scored a touchdown and an extra point, then the second team must do so in order for the overtime period to continue. If not, the game is over when the second team loses possession. If both teams score the same amount of points, a second round of overtime is played following another coin toss to determine who goes first. If a third overtime is needed, then the teams are forced to attempt a 2-point conversion following a touchdown. They cannot kick an extra point.

Since teams in American college football overwhelmingly prefer to go second [11,1], it is ex-post fairer to let teams bid who goes second, rather than let a coin toss determine this outcome. Teams could then submit bids as to the distance, beyond the 25 yard line, they are prepared to start from, if they go second. For example, if Team A and Team B submit bids b_A and b_B, respectively, and $b_A > b_B$, then Team A goes second, starting, in a first-price auction, from the $25 + b_A$ yard line, while Team B goes first, and starts from the 25 yard line. As in the NFL, the team that submitted the losing bid strictly prefers that the team submitting the winning bid will start further away from its goal line, which implies that the auction as to who goes second in overtime in college football is an auction with externality discussed above.

Overtime rules in the Canadian Football League (CFL) are similar to those in American college football (with one of the differences being that teams start from the 35 yard line rather than the 25 yard line). Teams prefer to go second rather than first in overtime and a resolution who goes first can be determined by bidding in an auction with externality of the type described above.

Overtime in chess

In the 2010 US Chess Championship Competition, the final stage of the regular competition between the last two contestants was a draw, and the players moved to an "Armageddon" Stage. In the Armageddon Stage, the player using the black pieces only has to end the game in a draw to win, while the player using the white pieces has to earn an outright victory. If one of the players needs a draw to win, they can play more conservatively. It makes them a heavy favorite even with black pieces. To decide who gets which side, the players bid time off their sixty-minute game clock. In the 2010 US Chess Championship Competition, the two remaining contestants were Yury Shulman and Gata Kamsky. Without knowing how much time their opponent was willing to sacrifice, Shulman bid 21 min off the clock and Kamsky offered 35 min. A first-price auction mechanism was implemented by the organizers, and accordingly, it left Kamsky with just 25 min to complete every move in the game as black versus the sixty minutes that Schulman had to complete his moves with the white pieces. The game ended in a draw, and Kamsky won (see, e.g., [5]).

Clearly, bidding to play black for a draw in the Armageddon Stage in chess is an auction with externality of the type considered above.

Overtime in soccer

Penalty shootouts in soccer, known as kicks from the penalty mark, which is located 11 m from the goal line, usually occur in knock-out soccer tournaments or cup competitions. After 90 min or extra-time, when the two teams are still tied, they move to the