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# Option pricing under jump-diffusion models with mean-reverting bivariate jumps

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ABSTRACT

We propose a jump-diffusion model where the bivariate jumps are serially correlated with a mean-reverting structure. Mathematical analysis of the jump accumulation process is given, and the European call option price is derived in analytical form. The model and analysis are further extended to allow for more general jump sizes. Numerical examples are provided to investigate the effects of mean-reversion in jumps on the risk-neutral return distributions, option prices, hedging parameters, and implied volatility smiles.

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## 1. Introduction

In options pricing, the jump-diffusion model is a popular extension from the diffusion model for the underlying stock price process. Earlier work can be dated back to Merton's classical model [7], and there have been a few variations proposed more recently, such as [1,5,6]. In these models, the risk-neutral dynamics of the stock price $S_t$ is assumed to follow

$$\frac{dS_t}{S_t} = (r - q - \lambda k)dt + \sigma dW_t + dJ_t, \qquad (1)$$

where $r, q$ are risk-free rate and dividend yield, $W_t$ is a Brownian motion, $J_t = \sum_{j=1}^{N(t)} (Y_j - 1)$ is a compound Poisson process with *i.i.d.* random jump ratio $Y_j$ and $k = \mathsf{E}[Y_j] - 1$ is the mean jump size. The Poisson process $N(t)$ with jump arrival rate $\lambda$ records the number of jumps happening in the time interval $[0, t]$. The stock price at time $t$ can also expressed as $S_t = S_0 e^{(r-q-\frac{\sigma^2}{2}-\lambda k)t+\sigma W_t + X_t}$, where $X_t = \sum_{j=1}^{N(t)} \ln Y_j$ is another compound Poisson process (as opposed to $J_t$). The variations in the above-mentioned studies differ in their assumptions on the distribution of $\ln Y_j$. For example, in Merton's model [7], $\ln Y_j$ follows a normal distribution. In the model considered by Amin [1] and Gukhal [5], $\ln Y_j$ follows a bivariate distribution meaning that it is either $+\delta$ or $-\delta$. Kou [6] proposed to assume that $\ln Y_j$ follows a double exponential distribution to reflect its leptokurtic feature. In these main stream jump-diffusion models, the jump process $J_t$ or $X_t$ is time homogeneous and has no serial correlation because both the underlying Poisson process and the jump size distribution are time homogeneous (with a fixed arrival rate $\lambda$ and a fixed distribution in $\ln Y_j$).

In view of this lack, this paper intends to consider an extension of the bivariate jump-diffusion (BJD) model of [1,5] such that the jump process is serially correlated. The serial correlation is introduced by assuming that it is less likely for the process to see the next jump going toward the same direction as the previous jump. If a positive (negative) jump has happened, then the probability of seeing a further positive (negative) jump will become smaller. This is motivated by the empirical observations that markets tend to overreact to unexpected events and the effect from a jump-causing event tends to become weaker as time goes by (see the empirical studies such as [3,2,4]). This makes the probability of seeing a sequence of positive (negative) jumps smaller than the corresponding probability in a time homogeneous model where the successive jumps follow *i.i.d.* distributions.

To this end, we construct a model termed mean-reverting bivariate jump-diffusion (MR-BJD) model which nests the BJD model as a special case. The mean-reverting property can be seen in the

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jump accumulation process $X_t$ in that it tends to be pulled back to its mean value when it moves away. We investigate the mathematical properties of the jump accumulation process in the MR-BJD model, and link it to the discrete OU process [8] so that we may use the results of the latter process to conduct our analysis.

The analysis of the jump accumulation process $X_t$ yields some analytical results that are useful in the subsequent derivations of the option pricing formulas for the proposed MR-BJD model. These formulas are obtained in analytical form which makes their calculations a simple task. To incorporate more general jump size distribution, we consider an extended version of the MR-BJD model, termed the MR-BNJD model, and provide its full analysis. A series of numerical examples are provided to examine the effects from the mean-reverting property on the risk-neutral distributions, option prices and hedging parameters. In addition, our numerical examples also demonstrate how the mean-reversion in jumps affects the shape of volatility smiles.

## 2. Modeling mean-reverting bivariate jumps

In this section we generalize the bivariate jump-diffusion (BJD) model such that the jumps are serially correlated with a mean-reverting structure. In the original BJD model as proposed in [1,5], the successive jump sizes are assumed to follow a bivariate distribution as below

$$\ln Y_j = \begin{cases} +\delta, & \text{with probability } p, \\ -\delta, & \text{with probability } 1-p. \end{cases}$$

The mean jump size (mean return caused by a jump) is $k = \mathsf{E}[Y] - 1 = pe^{\delta} + (1-p)e^{-\delta} - 1$. The compound Poisson process $X_t = \sum_{j=1}^{N(t)} \ln Y_j$ measures the accumulated contribution from the jumps occurring up to time $t$. Clearly, $X_t$ takes values from $\{\ldots, -2\delta, -\delta, 0, +\delta, +2\delta, \ldots\}$. The characteristic function of $X_t$ can be obtained easily as $\mathsf{E}[e^{i\alpha X_t}] = e^{\lambda t[pe^{i\alpha\delta}+(1-p)e^{-i\alpha\delta}-1]}$.

With $k$ defined above, it is not difficult to check that in the following BJD model

$$S_t = S_0 e^{\left(r-q-\frac{\sigma^2}{2}\right)t-\lambda kt+\sigma W_t+X_t}, \tag{2}$$

the martingale condition $\mathsf{E}[S_t] = S_0 e^{(r-q)t}$ holds and this ensures the absence of arbitrage (since $\mathsf{E}[e^{X_t}] = e^{\lambda kt}$). The call option price can be obtained by the conditioning technique (conditional on $N(t)$ and the number of up jumps; see [1,5]) as

$$C_{\mathrm{BJD}} = \sum_{n=0}^{\infty}\sum_{i=0}^{n}\left[e^{-\lambda t}\frac{(\lambda t)^n}{n!}\binom{n}{i}p^i(1-p)^{n-i} \times C_{\mathrm{BS}}(S_0 e^{-\lambda kt+(2i-n)\delta}, K, r, q, \sigma, t)\right], \tag{3}$$

where $C_{\mathrm{BS}}(S_0, K, r, q, \sigma, t) = S_0 e^{-qt}N(d_1) - Ke^{-rt}N(d_2)$, $d_1 = \frac{\ln\left(\frac{S_0}{K}\right)+(r-q+\frac{\sigma^2}{2})t}{\sigma\sqrt{t}}$, $d_2 = d_1 - \sigma\sqrt{t}$ is the Black–Scholes formula for call option price.

In this BJD model, since $\{\ln Y_j\}$ is an *i.i.d.* random sequence, the jump accumulation process $X_t$ is actually an asymmetric random walk. In the MR-BJD model, we intend to introduce serial correlation to $\{\ln Y_j\}$ in order to reflect the decaying momentum of successive jumps. Suppose that the $j$th jump is an up jump with probability $p$ and a down jump with probability $1-p$. The probability of seeing an up or down jump in the $(j+1)$-th jump is no longer the same, but depends on the outcome of the $j$th jump in the following way:

If $\ln Y_j = \pm\delta$, then

$$\ln Y_{j+1} = \begin{cases} +\delta, & \text{with probability } p \mp \xi; \\ -\delta, & \text{with probability } 1-p \pm \xi. \end{cases} \tag{4}$$

In other words, if the $j$th jump turns out to be an up (down) jump, then the probability of seeing a further up (down) jump will be decreased by $\xi$. It is clear that, with the above type of serial correlation, the jump accumulation process $X_t$ tends to be pulled back toward a central (mean) value when it goes far away. The MR-BJD model is the stock price model (2) where $X_t$ has such type of mean-reverting property, and the additional parameter $\xi$ measures the strength of mean-reversion.

We now provide an analysis on the jump accumulation process $X_t$ because it serves as the basis of the MR-BJD model. Let $p^s$ be the starting up jump probability (the first jump will be an up jump with probability $p^s$). Apparently, the successive up jump probabilities will be $p = p^s + i\xi$ where $i$ is an integer. For convenience, we assume the up and down jump probabilities take values from the following (identical) sets

$$\text{up jump probability } p \in \{1, 1-\xi, \ldots, 0.5, \ldots, \xi, 0\},$$

$$\text{down jump probability } 1-p \in \{0, \xi, \ldots, 0.5, \ldots, 1-\xi, 1\},$$

and $p^s$ belongs to these sets. Note that 1 and 0 also belong to these sets, and this ensures the legitimacy of the model (avoid yielding a negative or greater-than-one probability). These up (down) jump probabilities form a decreasing (increasing) arithmetic sequence, starting with 1 (0) and ending with 0 (1). In addition, 0.5 also belongs to these sets so that there is a case where up and down jumps are equally likely to happen.

Let the up probability takes a general form ($\ell$ is the index of the $2m+1$ values)

$$p_\ell = 1 - \ell\xi, \quad \ell = 0, \ldots, 2m,$$

where $\xi = \frac{1}{2m}$. The starting probability $p^s$ is one of them, i.e. $p^s = 1 - \ell\xi$ for some $\ell$. The jump accumulation process $X_t = \sum_{j=1}^{N(t)} \ln Y_j$ is then a process of finite states taking values from $\mathcal{X} = \{x_0, \ldots, x_m, \ldots, x_{2m}\} = \{\theta - m\delta, \ldots, \theta, \ldots, \theta + m\delta\}$, or

$$x_\ell = \theta - (m-\ell)\delta, \quad \ell = 0, \ldots, 2m.$$

The central state value $\theta$ is the mean value of $\mathcal{X}$ corresponding to the case where $p = 1-p = 0.5$. Note that $x_\ell = 0$ for some $\ell$ because $X_t$ starts with $X_0 = 0 \in \mathcal{X}$. Each time when a jump happens, $X_t$ moves up or down by $\delta$. One significant difference from the BJD case is that the domain of the $X_t$ in BJD is unbounded (infinite states) but the domain of the $X_t$ in MR-BJD is bounded (finite states, because the probability of jumping beyond $[x_0, x_{2m}]$ is zero). This can also be seen from the number of states $2m+1 = \frac{1}{\xi}+1 \to \infty$ as $\xi \to 0$. The mean-reverting property is observed from the greater probabilities of pulling $X_t$ back toward $\theta$ when it moves farther away from it.

Fig. 1 shows two possible scenarios of the process $X_t$. We see that $\xi$ represents the strength of mean-reversion. Starting with the same probability set $(p^s, 1-p^s) = (0.6, 0.4)$, (a) shows the stronger mean-reversion case with $\xi = 0.1$ while (b) shows the relatively weaker case with $\xi = 0.05$. The mean value $\theta$ depends on $\xi$ by the following formula

$$\theta = \frac{(p^s - 0.5)\delta}{\xi}$$

which equals $\delta$ in (a) and $2\delta$ in (b).

Note that the jump accumulation process $X_t$ is a continuous-time Markov chain (CTMC). At each state value $x_\ell \in \mathcal{X}$, it may either jump up to state $x_{\ell+1}$ with a transition rate $u_\ell$ or jump down to state $x_{\ell-1}$ with rate $d_\ell$. Because the jumps are driven by the Poisson process $N(t)$ whose arrival rate is $\lambda$, the up and down transition rates for $X_t = x_\ell$ are given by

$$u_\ell = \lambda p_\ell = \lambda(1-\ell\xi), \qquad d_\ell = \lambda(1-p_\ell) = \lambda(\ell\xi). \tag{5}$$

The process $X_t$ with parameter set $(\lambda, \delta, p^s, \xi)$ is now well defined as a CTMC.

It is worth discussing the analogue relation between the CTMC $X_t$ and its corresponding diffusion process $\bar{X}_t$. In the original BJD case, the jump accumulation process $X_t$ has a fixed tendency to

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