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Optimal quality provision when reputation is subject to random inspections

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ABSTRACT

We consider a firm whose profit is determined by its reputation. The quality of its products is unobservable, but random inspections reveal the true quality and change the reputation. We obtain closed-form solutions for the provision of quality and show that increasing the inspection rate can be disadvantageous for customers.

1. Introduction

We consider a firm whose reputation is related to the quality of the products it offers. The quality is in general not observable by customers, at least not before purchasing the product. Therefore, the customers' buying decision depends – apart from price – crucially on the reputation of the firm. The main mechanism for determining the reputation is random inspections. An authority conducts inspections which reveal the true quality. If the true quality deviates from the current reputation, then the reputation is immediately changed. The main questions are: how does a firm behave under such a framework? Which level of quality does the firm offer and how does its reputation evolve? And what are the implications for a regulator that wants to set an optimal inspection rate?

In the paper, we focus on two different types of products and quality, respectively: *credence goods* and *experience goods*. For credence goods [2], quality relates to the production process. At no point can the customer observe the quality of the product, not even after having consumed it. We think, e.g., of products with the Fairtrade certification or organic production. Customers cannot observe the production process. Their only reference point is the reputation of the firm. By random inspections, an authority (e.g., consumerism) can determine the "quality" of the production process.

Conversely, the quality of experience goods [6] can be observed by customers *after* having consumed it. Examples include meals in restaurants, hotels and medical services. Again, we assume that reputation is determined by random inspections by authorities, consumerism or reviewers. Additionally, consumers can share their experience by word-of-mouth. This *diffusion of information* influences the reputation continuously.

We find that for credence goods the firm offers a constant level of quality. This level is independent of the firm's initial reputation. The firm raises the quality level if the rate of inspections is increased, but the marginal increase in quality diminishes. With this knowledge, a regulator can trade off inspection costs against an increased level of quality offered by the firm.

For experience goods, the quality level offered still converges to a constant level in the long run. In the short run, however, the firm offers a varying quality level below or above the long-run level. The offered level is inverse to its reputation (high reputation yields low quality and vice versa) and is adjusted continuously. Therefore, reputation may be misleading since it deviates from the true quality level. However, in time, the deviation decreases. From the viewpoint of a regulator, we show that in some cases a lower inspection rate is better than a higher inspection rate, and that this depends in particular on the initial reputation of the firm.

The model is formulated as a stochastic optimal control problem. Our concept of reputation is similar to the stock of goodwill in the Nerlove–Arrow model [7] which is still widely used, e.g., [5]. The difference is that in the Nerlove–Arrow model, the capital stock is built up and depreciates continuously and deterministically, while in our model the capital stock can exhibit random discontinuities due to inspections.

The inspections are modeled as a Poisson jump process. The jumps correct the difference between the reputation and the actual quality and, in some sense, reveal the otherwise hidden behavior

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of the firm. Although optimal control problems with jumps "are a standard tool in the economic literature" [9], they are rarely used to repeatedly reveal states that can be changed continuously. In particular, to the best of our knowledge, this is the first time that random inspections are treated in such a framework.

There is a second component in the model dynamics, diffusion of information, that is widely used in economics, e.g., [4]. For an optimal control setting, many different models were proposed; for an early overview see [11]. The actual type of dynamics and the analysis of how information diffuses do not lie at the center of our attention. We assume that the true quality of the product is revealed continuously in time. Our simple but very intuitive dynamics allow for a closed-form solution.

The structure of the paper is as follows. In Section 2, we state the model and the assumptions. In Section 3, we formulate deterministic variations of the model without inspections, which serve as benchmarks. In Section 4, we consider only random inspections which correspond to credence goods. In Section 5, diffusion of information is added. This model corresponds to experience goods. Section 6 presents ideas for future research.

2. Model formulation

Consider a firm and its *reputation*, which is expressed in terms of the publicly *perceived* level of quality Q. The revenue of the firm $\hat{\Pi}(Q, p)$ depends upon reputation and price p. By assuming that the price for each level Q is set accordingly as to maximize revenues, we can focus on reputation only. Hence, the revenue function of the firm is $\Pi(Q) = \max_p \hat{\Pi}(Q, p)$. Independent of its reputation, the firm may choose to offer a different level of quality q. This *offered* or *true* quality incurs costs $\hat{c}(q)$ on the firm. In general, offered quality is not observable by customers. It is revealed by stochastic, Poisson-distributed inspections and possibly by diffusion of information. Offering a lower level of quality saves the firm money instantaneously but, if revealed, lowers the reputation Q. We denote the costs of offering the quality level q by $\hat{c}(q)$. Let r be the discount rate. The firm is risk-neutral and maximizes its expected profit:

$$\max_q \mathbb{E}\left[\int_0^\infty (\Pi(Q) - \hat{c}(q))e^{-rt}\, dt\right], \tag{1}$$

subject to the dynamics

$$dQ = \delta(q - Q)\, dt + (q - Q)\, dJ. \tag{2}$$

As indicated above, the dynamics allow for two possibilities of revelation of the true level of quality q. The deterministic term corresponds to diffusion of information. The difference between the true level q and reputation Q is observed after the consumption of the good and the information is spread at rate δ. The stochastic term, dJ, corresponds to a random inspection and denotes the increments of a Poisson process with arrival rate λ. At random times, the Poisson process jumps and the increment equals 1. This immediately corrects the state Q for the difference between the true level q and the perceived level Q. In between jumps, the stochastic term does not contribute to the dynamics.

We assume that the revenue function is quadratic and concave in state Q, $\Pi(Q) = -p_2/2\ Q^2 + p_1 Q$. Due to the form of the revenue function we place an upper bound upon the control $q \leq \overline{q} = p_1/p_2$. This is done to avoid an increase in reputation causing a decrease in revenue: as every control can become the actual state, $q > p_1/p_2$ would imply $\Pi(q)' > 0$. Costs are assumed to be linear and increasing in the control q, $\hat{c}(q) = cq$. These assumptions allow for a closed-form solution.

3. Deterministic cases

The following deterministic models without inspections serve as benchmark models. For the models to have a solution, we have to restrict the feasible controls, $q \in [\underline{q}, \overline{q}]$.

3.1. Static case

Proposition 1. *If the true quality level is observable, then the optimal strategy is to offer a constant quality level*

$$q^S = \frac{p_1 - c}{p_2}. \tag{3}$$

If the true quality level is never observable, then the optimal strategy is to offer the lowest possible level $q = \underline{q}$.

Proof. Since we are in a deterministic setting, we can omit the expectation in the maximization problem (1). If the true quality level is observable, then the problem is equivalent to the following maximization problem:

$$\max_{q\in[\underline{q},\overline{q}]} \int_0^\infty \left(\frac{-p_2}{2} q^2 + p_1 q - cq\right) e^{-rt}\, dt,$$

where q is the level of quality offered. Pointwise maximization yields the claimed result.

On the other hand, if the true quality level is not observable at all, it is clearly optimal to provide the lowest possible quality level $q = \underline{q}$. This minimizes the costs and maximizes the profit. □

We refer to the solution in (3) as the *complete information solution.*

3.2. Dynamic case—diffusion of information

The proposition above describes the behavior of the firm if offered quality is always observable or if it is never observable. In some cases, the true quality level q is not observable at first but can be experienced after consumption. The experience is shared by word-of-mouth and the information diffuses. The maximization problem is (1) – omitting the expectation – and the dynamics of diffusion are given by the deterministic part of (2).

Proposition 2 (*Diffusion-Only*)**.** *Assume that the information about the difference between reputation Q and the true quality level q diffuses at rate δ. Then the optimal strategy is a bang–bang type strategy that approaches a constant level as fast as possible and remains at this level for all times:*

$$q^*(Q) = \begin{cases} \underline{q}, & \text{for } Q > q^\delta \\ q^\delta, & \text{for } Q = q^\delta \\ \overline{q}, & \text{for } Q < q^\delta \end{cases}$$

with

$$q^\delta = \frac{p_1 - \left(1 + \frac{r}{\delta}\right) c}{p_2}.$$

Proof. For a proof follow, e.g., [3, Section 3.3]. □

The economic interpretation of the solution is that for a high initial level of perceived quality the firm can exploit its reputation. It saves costs by offering the lowest possible quality for some time while reputation decreases. This is done until reputation reaches the level q^δ. Conversely, starting with a low initial level, the firm wants to increase reputation. This is costly, and the increase in reputation is sluggish. Reaching the complete information solution is too costly for the firm, $q^\delta < q^S$ for all $\delta \geq 0$, although $q^\delta \to q^S$, for $\delta \to \infty$.

4. Random inspections only

Now we turn to random inspections and credence goods. We assume that an authority has the power to inspect the firm and reveal the difference between the reputation and the true behavior. As an