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# Optimization of demand response through peak shaving

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ABSTRACT

We consider a consumer of a resource, such as electricity, who must pay a per unit charge to procure the resource, as well as a peak demand charge. We will assume that this consumer has some ability to self-generate and present an efficient linear programming formulation for the demand response of such a consumer. We will establish a monotonicity result that indicates fuel supply of $S$, utilized for self-generation, may be spent in successive steps adding to $S$.



## 1. Introduction

We consider a model in which a consumer of a resource over several periods must pay a per unit charge for the resource as well as a peak usage charge. The consumer has the ability to reduce consumption in any period at some given cost, subject to a constraint on the total amount of reduction possible. The consumer's problem is to decide in what periods to reduce consumption to minimize the total cost of procuring the resource.

Such a model could arise in several settings. We have in mind an industrial or commercial consumer of electricity who uses a varying amount of electricity over some time horizon of $\mathcal{T}$ periods, for which he or she incurs an energy charge (per megawatt-hour consumed) and a peak usage charge for the total megawatt-hours consumed in the highest $k$ periods. The peak usage charge is otherwise known as a demand charge. The consumer has some onsite local generation that can be used to offset the purchases of electricity in any period. Such a charging regime is called *anytime peak pricing* or "Hopkinson rate" after the engineer who first proposed it in 1892 (see [3]).

A much simplified version of our problem was addressed in the late 1970s and early 1980s, before the prevalence of electricity markets, in the context of public utility pricing and rationing when demand exceeds the available supply (see for instance [1,5,4]). In this context, the authors attempt to deal with the details of rationing by assuming aggregate infinitesimal consumers that would provide a simple elastic demand curve with no constraints. This is a large point of difference from the setting that we face, where our consumer, possibly due to manufacturing constraints, is inflexible with respect to consumption of electricity. Furthermore, the above authors do not study properties (such as monotonicity) of their models.

Anytime peak pricing can be contrasted with *coincident peak pricing* (and its relation "time-of-use" pricing) which imposes a demand charge in periods when the *system* experiences peak demand (as modeled in [6] for example). The Hopkinson rate was originally intended to charge for electricity when it was primarily used for lighting, and so any user's peak demand typically coincided with the system peak. When these are different, it is not hard to see that coincident peak charging provides a clearer incentive to reduce the system costs incurred by increases in capacity. Notwithstanding this, anytime peak pricing does provide benefits from peak reduction (see e.g. [7]). It is also worth mentioning that for geographically isolated customers, coincident peak reduces to the Hopkinson rate.

Although the problem for a consumer facing an anytime peak charge is more straightforward than tackling the coincident peak problem, it is not trivial. The peak charge will typically be made on the total consumption over several periods, typically those $k$ periods with the largest consumption over some predetermined horizon. In this paper we show how these periods can be determined by a linear programming problem, to give an overall problem of minimizing cost that is also a linear program. This linear program is then shown to satisfy a monotonicity property that makes it amenable to solution by a greedy algorithm. This provides some insights into how to attack the problem with random data.

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Although our problem might have applications in other settings we will couch it in the setting of electricity procurement. Nevertheless the analysis we develop is quite general.

The paper is laid out as follows. In the next section we formulate the optimization problem we will study, and show that it simplifies to a linear program. In Section 3 we show that this linear program has a specific structure that enables its solution by a greedy algorithm. The algorithm is outlined in Section 4; details can be found in [2].

## 2. The anytime peak demand problem

We start by defining the parameters and the variables of the problem. Throughout, we measure electricity in terms of the units of the fuel needed to produce it.

*Parameters*

$\mathcal{T}$ = set of periods.
$d_t$ = demand in period $t$.
$p_t$ = spot price in period $t$.
$c_t$ = cost of generating one unit of electricity using fuel in period $t$.
$a_t$ = safe operating capacity of the generator in period $t$. For the sake of simplicity, we make the assumption that $a_t \leq d_t$ (avoiding the case of "selling back to the grid").
$S$ = total fuel supply.
$P$ = the peak demand charge.
$k$ = the number of periods to which the maximum demand charge applies.

*Variables*

$s_t$ = amount of fuel to allocate to generation in period $t$.
$M$ = sum of the largest $k$ load realizations.

The consumer's problem over a time horizon $\mathcal{T}$ is to minimize the total cost of electricity consumed, plus the peak charges that are incurred on the top $k$ periods, while meeting every period's demand and employing a limited amount of self-generation. Without loss of generality we assume that $|\mathcal{T}| \geq k$. This problem can be formulated as:

$$
\begin{aligned}
&[\text{AP}]: \min PM + \sum_{t\in\mathcal{T}} (c_t - p_t)s_t \\
&\text{s.t.} \sum_{t\in\mathcal{T}} s_t \leq S \\
&s_t \leq a_t \qquad t \in \mathcal{T} \\
&\sum_{t\in\tau} (d_t - s_t) \leq M \qquad \text{for all } \tau \subseteq \mathcal{T},\ |\tau| \leq k.
\end{aligned}
$$

Note that there is no loss in generality in assuming that $P = 1$ (by scaling the objective function of [AP]), so we normalize the peak demand charge, i.e., set $P = 1$, simply to make the presentation clearer.

Observe that in [AP] all subsets of $\mathcal{T}$ of size $k$ or less must be included which gives an exponentially growing set of constraints. The problem [AP] can be formulated more concisely using the following observation.

Given any feasible solution $s_t,\ t \in \mathcal{T}$, for [AP], the cost of maximum demand $M$ is the optimal value of

$$
\begin{aligned}
&[\text{MDP}]: \max \sum_{t\in\mathcal{T}} \lambda_t\,(d_t - s_t) \\
&\text{s.t.} \sum_{t\in\mathcal{T}} \lambda_t = k, \qquad [h] \\
&\lambda_t \leq 1 \qquad t \in \mathcal{T} \qquad [y_t] \\
&\lambda_t \geq 0 \qquad t \in \mathcal{T}.
\end{aligned}
$$

Taking the dual of MDP gives

$$
\begin{aligned}
&[\text{MDD}]: \min kh + \sum_{t\in\mathcal{T}} y_t \\
&\text{s.t.} h + y_t \geq (d_t - s_t) \qquad t \in \mathcal{T} \qquad [\lambda_t] \\
&y_t \geq 0 \qquad t \in \mathcal{T}
\end{aligned}
$$

which has the same optimal value $M$. Here $M$ is the sum of the residual demands $d_t - s_t$ over the $k$ highest periods, which incurs penalty 1. Henceforth we write $\forall t$ instead of $t \in \mathcal{T}$ for short.

If [MDP] and [MDD] have multiple optimal solutions we need to focus on particular optimal solutions. Let us define $g(k)$ to be the $k$th largest value of $d_t - s_t$ for $t \in \mathcal{T}$. We will then construct a set of periods that constitute the top $k$ periods (in terms of $d_t - s_t$), by resolving some ties. Define $\mathcal{N} = \{t | d_t - s_t > g(k)\}$. Now consider the set $\{t | d_t - s_t = g(k)\}$, order this set by $t$, and select the elements of $\mathcal{O}$ to be the first $k - |\mathcal{N}|$ periods in this (ordered) set. We will define $\mathcal{M} = \mathcal{N} \cup \mathcal{O}$. Note that $|\mathcal{M}| = k$, so we have determined a way of selecting "the top $k$ periods of residual demand" without ambiguity. We refer to $\mathcal{M}$ as our *canonical maximum demand* set. Note also that $\mathcal{M}$ depends on the vector $d - s$.

**Lemma 1.** *For a given vector $d - s$, optimal solutions to [MDD] and [MDP] are given by*

$$
\begin{aligned}
&h^* = g(k) \\
&y_t^* = \max(d_t - s_t - h^*, 0) \quad \forall t, \ \textit{and} \\
&\lambda_t^* = \begin{cases} P, & \textit{if } t \in \mathcal{M}, \\ 0, & \textit{otherwise}. \end{cases}
\end{aligned}
$$

We refer to these solutions as the canonical solutions for residual demand $d - s$.

**Proof.** Observe that

$$
y_t^* = \begin{cases} d_t - s_t - h^*, & \text{if } t \in \mathcal{M}, \\ 0, & \text{otherwise}. \end{cases}
$$

Thus the optimality conditions for [MDP] and [MDD],

$$
\begin{aligned}
&\sum_{t\in\mathcal{T}} \lambda_t = k \\
&0 \leq \lambda_t \leq 1 \quad \forall t \\
&h + y_t \geq d_t - s_t \quad \forall t \\
&y_t \geq 0 \quad \forall t \\
&y_t(1 - \lambda_t) = 0 \quad \forall t \\
&\lambda_t(h + y_t + s_t - d_t) = 0 \quad \forall t,
\end{aligned}
$$

are satisfied by the solution in the statement of the lemma. Hence we have optimal solutions. ■

It is worth noting that [MDD] will almost always have an infinite number of solutions of which the canonical solution is only one. In fact for any $0 \leq \alpha \leq 1$,

$$
\begin{aligned}
&h^*(\alpha) = \alpha g(k) + (1 - \alpha) g(k+1), \\
&y_t^*(\alpha) = \max(d_t - s_t - h^*(\alpha), 0) \quad \forall t, \ \text{and} \\
&\lambda_t^* = \begin{cases} 1, & \text{if } t \in \mathcal{M}, \\ 0, & \text{otherwise}, \end{cases}
\end{aligned}
$$

will satisfy the optimality conditions of [MDP] and [MDD] and are therefore optimal.

Following [MDP] and [MDD], we can formulate [AP] as a linear program without having to consider an exponentially growing set