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A note on supplier diversification under random yield

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ABSTRACT

Fadıloğlu et al. and Tajbakhsh et al. consider supplier diversification in an EOQ type inventory setting with multiple suppliers and binomial yield and show that working with a single supplier is always optimal. In this short note, we present an alternative and more elegant proof of the optimality of sole sourcing in the EOQ model with general random yield.

1. Introduction

Fadıloğlu et al. [6] and Tajbakhsh et al. [13] consider supplier diversification in an EOQ type inventory setting with multiple independent suppliers and binomial yield. They show that, under mild condition, sole sourcing is always optimal. In this note, we present an alternative and more elegant proof of the optimality of sole sourcing in the EOQ model with general random yield. Our approach extends this result to a general structure for the positively correlated random yield and provides more insights into the problem. Furthermore, our results extend those of [6,13] in the following two directions:

(1) the form of the random yield in this note is more general than the random yield used in [6,13]. That is, the problem studied in [6,13] is a special case of this note.
(2) when the fixed ordering cost is separable for each supplier, it is optimal for the system to work with single supplier under positively correlated random yield.

The wide existence of yield uncertainty has drawn significant attention of scholars and turns the random yield to an important research topic in the area of operations management and production economics (e.g., [2,7,6,5,4,3,8,10,13]). We refer the reader to the survey papers of [14,12] for a comprehensive review of the literature. Positively correlated random yield exists between firms that rely on the same production inputs or among companies that rely on the same geographical market. If two suppliers source a major component from a common second-tier supplier (especially a large one), their yields can be positively correlated as both depend on the quality of the component offered by the common supplier. Many examples of positive default correlation can be found in [9,11].

We consider an EOQ model in which orders are placed with n ($n \leq N$) suppliers, where N denotes the total number of available suppliers. Let ℓ denote the set of the n chosen suppliers indexed by i and S denote the set of the N available suppliers, i.e., ℓ is a nonempty subset of S. Without loss of generality, we assume that ℓ is composed of the first n suppliers in S, i.e., $\ell = \{i|1 \leq i \leq n\} \subseteq S$. Let $|\ell|$ denote the cardinality of set ℓ, i.e., $|\ell| = n$ and $|S| = N$. The constant demand rate is D and the unit holding cost per unit time is h. No backorders are allowed. We assume that the replenishments are instantaneous (i.e., zero lead time) and the random yield is described as follows: an ordering of q_i ($q_i > 0$) units from supplier i ($i \in \ell$) will receive $\alpha_i(q_i)$ units of the product, where $\alpha_i(q_i)$ is a non-negative random variable which is dependent on q_i. Denote $\mu_i(q_i)$ and $\sigma_i(q_i)$ as the mean and standard deviation of $\alpha_i(q_i)$, respectively. We assume that $\alpha_i(q_i)$ and $\alpha_j(q_j)$ ($i \neq j$) may be positively correlated or independent with correlation coefficient $\rho_{ij}(q_i, q_j) \geq 0$ for any q_i and q_j, where $\rho_{ij}(q_i, q_j) = 0$ indicates $\alpha_i(q_i)$ and $\alpha_j(q_j)$ are *uncorrelated*.

Let c_i denote the unit variable cost of the item received from supplier i. We assume that c_i is the cost per non-defective unit. This assumption has been used by many researchers (e.g., [3–5,13]). Let $K(\ell)$ be the fixed ordering cost when the order is split among all the suppliers in ℓ (i.e., $q_i > 0$ for any $i \in \ell$). Unless stated otherwise, the structure of $K(\ell)$ is completely arbitrary. For ease of exposition, we denote $K_i = K(\{i\})$ as the fixed ordering cost when the order is placed only with supplier i ($i \in \ell$). Following expression (6) in [13], we also assume $K_i \leq K(\ell)$ for any $i \in \ell$.

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2. Optimality of single-sourcing

Note that $E[\alpha_i^2(q_i)] = \mu_i^2(q_i) + \sigma_i^2(q_i)$ and $E[\alpha_i(q_i)\alpha_j(q_j)] = \mu_i(q_i)\mu_j(q_j) + \rho_{ij}(q_i, q_j)\sigma_i(q_i)\sigma_j(q_j)$, we have

$$
\begin{aligned}
E\left[\sum_{i=1}^{n} \alpha_i(q_i)\right]^2
&= \sum_{i=1}^{n} E\left[\alpha_i^2(q_i)\right] + 2 \sum_{1 \le i < j \le n} E\left[\alpha_i(q_i)\alpha_j(q_j)\right] \\
&= \sum_{i=1}^{n}\left[\mu_i^2(q_i) + \sigma_i^2(q_i)\right] + 2 \sum_{1 \le i < j \le n}\left[\mu_i(q_i)\mu_j(q_j) + \rho_{ij}(q_i, q_j)\sigma_i(q_i)\sigma_j(q_j)\right] \\
&= \left[\sum_{i=1}^{n} \mu_i(q_i)\right]^2 + \sum_{i=1}^{n}\left[\sigma_i^2(q_i)\right] + 2 \sum_{1 \le i < j \le n}\left[\rho_{ij}(q_i, q_j)\sigma_i(q_i)\sigma_j(q_j)\right].
\end{aligned}
$$

Then the expected total cost per cycle is given by

$$
\begin{aligned}
K(\ell) &+ \sum_{i=1}^{n} c_i E[\alpha_i(q_i)] + \frac{h}{2D} E\left[\sum_{i=1}^{n} \alpha_i(q_i)\right]^2 \\
&= K(\ell) + \sum_{i=1}^{n} c_i \mu_i(q_i) + \frac{h}{2D}\left\{\left[\sum_{i=1}^{n} \mu_i(q_i)\right]^2 + \sum_{i=1}^{n}\left[\sigma_i^2(q_i)\right] + 2 \sum_{1 \le i < j \le n}\left[\rho_{ij}(q_i, q_j)\sigma_i(q_i)\sigma_j(q_j)\right]\right\}. \qquad (2.1)
\end{aligned}
$$

Denote the long-run average cost per unit time by $C(q_1, \ldots, q_n)$. By the Renewal Reward Theorem and using (2.1), we have

$$
\begin{aligned}
C(q_1, \ldots, q_n)
&= \frac{K(\ell) + \sum_{i=1}^{n} c_i E[\alpha_i(q_i)] + hE\left[\sum_{i=1}^{n} \alpha_i(q_i)\right]^2/(2D)}{\left[\sum_{i=1}^{n} \mu_i(q_i)\right]/D} \\
&= \frac{DK(\ell) + D\sum_{i=1}^{n} c_i\mu_i(q_i) + \frac{h}{2}\sum_{i=1}^{n} \sigma_i^2(q_i) + h\sum_{1 \le i < j \le n} \rho_{ij}(q_i, q_j)\sigma_i(q_i)\sigma_j(q_j)}{\sum_{i=1}^{n} \mu_i(q_i)} + \frac{h}{2}\left[\sum_{i=1}^{n} \mu_i(q_i)\right] \\
&\ge G(q_1, \ldots, q_n), \qquad (2.2)
\end{aligned}
$$

where

$$
G(q_1, \ldots, q_n) = \frac{DK(\ell) + D\sum_{i=1}^{n} c_i\mu_i(q_i) + \frac{h}{2}\sum_{i=1}^{n} \sigma_i^2(q_i)}{\sum_{i=1}^{n} \mu_i(q_i)} + \frac{h}{2}\left[\sum_{i=1}^{n} \mu_i(q_i)\right]. \qquad (2.3)
$$

The last inequality in (2.2) is due to $\rho_{ij}(q_i, q_j) \ge 0$ for any q_i and q_j. In the rest of this note, we consider two cases in which it is optimal to work with single supplier.

2.1. Linear variance

In this subsection, we denote $\widetilde{q}_i = \mu_i(q_i)$ and assume that $\widetilde{q}_i$ is strictly increasing in q_i. Note that there exists one-to-one correspondence between $\widetilde{q}_i$ and q_i, we will regard $\widetilde{q}_i$ as the decision variable and denote $q_i = \mu_i^{-1}(\widetilde{q}_i)$, where $\mu_i^{-1}(\cdot)$ is the inverse function of $\mu_i(\cdot)$. Then $\sigma_i(q_i) = \sigma_i(\mu_i^{-1}(\widetilde{q}_i))$. For notational simplicity, we define $\delta_i(\widetilde{q}_i) = \sigma_i(\mu_i^{-1}(\widetilde{q}_i))$. From (2.3), $G(q_1, \ldots, q_n)$ can be rewritten as

$$
\widetilde{G}(\widetilde{q}_1, \ldots, \widetilde{q}_n) = \frac{DK(\ell) + D\sum_{i=1}^{n} c_i\widetilde{q}_i + \frac{h}{2}\sum_{i=1}^{n} \delta_i^2(\widetilde{q}_i)}{\sum_{i=1}^{n} \widetilde{q}_i} + \frac{h}{2}\left(\sum_{i=1}^{n} \widetilde{q}_i\right). \qquad (2.4)
$$

If the system orders only from supplier i, then the long-run average cost with expected quantity $\widetilde{q}_i$ can be described as

$$
\widetilde{G}_i(\widetilde{q}_i) = \frac{DK_i + Dc_i\widetilde{q}_i + \frac{h}{2}\delta_i^2(\widetilde{q}_i)}{\widetilde{q}_i} + \frac{h}{2}\widetilde{q}_i. \qquad (2.5)
$$

Lemma 2.1. *If $\delta_i^2(\widetilde{q}_i) = A_i\widetilde{q}_i + B_i$ with $A_i, B_i \ge 0$, then it is better for the system to order only from supplier i_0 than to order from all the n suppliers in ℓ (i.e., $q_i > 0$ for all $i \in \ell$), where*

$$
i_0 \in L^* = \arg\min_{i \in \ell}\left\{Dc_i + \frac{h}{2}A_i\right\}. \qquad (2.6)
$$

Proof. Note that $\delta_i^2(\widetilde{q}_i) = A_i\widetilde{q}_i + B_i$, then from (2.4), we have

$$
\begin{aligned}
\widetilde{G}(\widetilde{q}_1, \ldots, \widetilde{q}_n)
&= \frac{DK(\ell) + D\sum_{i=1}^{n} c_i\widetilde{q}_i + \frac{h}{2}\sum_{i=1}^{n}(A_i\widetilde{q}_i + B_i)}{\sum_{i=1}^{n} \widetilde{q}_i} + \frac{h}{2}\left(\sum_{i=1}^{n} \widetilde{q}_i\right) \\
&= \frac{DK(\ell) + \sum_{i=1}^{n}\left[\left(Dc_i + \frac{h}{2}A_i\right)\widetilde{q}_i + \frac{h}{2}B_i\right]}{\sum_{i=1}^{n} \widetilde{q}_i} + \frac{h}{2}\left(\sum_{i=1}^{n} \widetilde{q}_i\right). \qquad (2.7)
\end{aligned}
$$

By the definition of L^* in (2.6), we have $Dc_{i_0} + \frac{h}{2}A_{i_0} \le Dc_i + \frac{h}{2}A_i$ for any $i_0 \in L^*$ and $i \in \ell$. If $|L^*| \ge 2$, then i_0 is any element of L^*. Since $A_i, B_i \ge 0$, we have

$$
\begin{aligned}
\sum_{i=1}^{n}&\left[\left(Dc_i + \frac{h}{2}A_i\right)\widetilde{q}_i + \frac{h}{2}B_i\right] \\
&\ge \left(Dc_{i_0} + \frac{h}{2}A_{i_0}\right)\sum_{i=1}^{n}\widetilde{q}_i + \frac{h}{2}\sum_{i=1}^{n} B_i \\
&\ge \left(Dc_{i_0} + \frac{h}{2}A_{i_0}\right)\sum_{i=1}^{n}\widetilde{q}_i + \frac{h}{2}B_{i_0}. \qquad (2.8)
\end{aligned}
$$

Combining (2.7) with (2.8), we have

$$
\widetilde{G}(\widetilde{q}_1, \ldots, \widetilde{q}_n) \ge \frac{DK(\ell) + \left(Dc_{i_0} + \frac{h}{2}A_{i_0}\right)\sum_{i=1}^{n}\widetilde{q}_i + \frac{h}{2}B_{i_0}}{\sum_{i=1}^{n} \widetilde{q}_i} + \frac{h}{2}\left(\sum_{i=1}^{n} \widetilde{q}_i\right)
$$