Operations Research Letters 36 (2008) 551–556

Contents lists available at ScienceDirect

Operations Research Letters

journal homepage: www.elsevier.com/locate/orl

Newsvendor equations for optimal reorder levels of serial inventory systems with fixed batch sizes

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ARTICLE INFO

Article history:
Received 15 February 2006
Accepted 3 June 2008
Available online 11 June 2008

Keywords:
Inventory
Multi-echelon
(R, nQ) policy
Newsvendor equations

ABSTRACT

We consider a stochastic serial inventory system with a given fixed batch size per stage and linear inventory holding and penalty costs. For this system, echelon stock (R, nQ) policies are known to be optimal. On the basis of new average costs formulas, we obtain newsvendor equations for the optimal reorder levels.

1. Introduction

We consider an N-echelon serial inventory/production system under periodic review and centralized control ($N \geq 2$). The most downstream stage, stage 1, orders from stage 2, 2 from 3, $\ldots$, $N-1$ from N, and stage N from an external supplier (called stage N +1) with ample stock. Any order of stage i is an integer multiple of a base order quantity Q_i. Further, we assume that $Q_{i+1} = n_i Q_i$ for $i = 1, \ldots, N-1$ where n_i is a positive integer. Because of this integer-ratio assumption, we assume that the initial on-hand stock at stage $i = 2, \ldots, N$ is an integer multiple of Q_{i-1}. Stage 1 faces stochastic demand of customers. Demands in different periods are i.i.d. Any unfulfilled customer demand at stage 1 is backlogged. Leadtimes are deterministic, and the costs consist of linear inventory holding and penalty costs. The objective is to minimize the average costs over an infinite horizon.

The system under study generalizes the basic serial Clark–Scarf system (cf. [4]). The fixed batch sizes are exogenous. They may be set at a higher planning level, where aspects such as capacity constraints (for multiple products), setup times and costs, and material handling constraints are taken into account. By the specification of fixed batch sizes, the numbers of setups and material handling activities are controlled. Alternatively, they may be controlled by fixed replenishment intervals (cf. [10]) or by a combination of the two (cf. [1]).

For the system with fixed batch sizes, Chen [3] has shown that an optimal ordering policy is to follow an echelon stock (R, nQ) policy at each stage: whenever the echelon inventory position at stage i is at or below the reorder point R_i, a minimum integer multiple of its base quantity (Q_i) that brings its inventory position above R_i is ordered from stage $i+1$. By Chen [2], optimal reorder points $R_1^*, \ldots, R_N^*$ are obtained by a sequential minimization of one-dimensional cost functions for so-called subsystems. Subsystem i is an i-stage serial system consisting of the stages $1, \ldots, i$ and with ample supply at stage $i+1$ and modified cost parameters. For $i = 1, \ldots, N$, the optimal reorder level R_i^* is obtained by the minimization of the average costs for subsystem i. The main contribution of our paper consists of the derivation of new formulas for these N one-dimensional cost functions, in terms of so-called expected shortfalls and backlogs, and newsvendor equations for the optimal reorder levels.

The newsvendor equations show a direct relationship between the probability of no-stockout at stage 1 and optimal reorder levels. Suppose optimal reorder levels $R_1^*, \ldots, R_i^*$ for the stages $1, \ldots, i$ have been determined. Then an optimal reorder level R_{i+1}^* is such that the no-stockout probability at stage 1 in subsystem $i+1$ with reorder levels $(R_1^*, \ldots, R_i^*, R_{i+1}^*)$ is equal to a specific newsvendor fraction. The newsvendor equations contribute to a better understanding of optimal control, and in particular of the optimal positioning of safety stock as a function of added values per stage.

In the literature, pure or generalized basestock policies (such as (R, nQ) policies) have been shown to be optimal for several multi-echelon systems: serial systems with fixed batch sizes, fixed replenishment intervals, advance demand information, and Markov

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doi:10.1016/j.orl.2008.06.003

modulated demand, and also assembly and distribution systems (for references, see [9]). We believe that for many, or maybe even all, of these systems newsvendor equations may be derived. So far, newsvendor equations have been derived for the serial Clark–Scarf system [11,9], serial systems with fixed replenishment intervals [10], and distribution systems under the balance assumption (Diks and De Kok [5]). This study extends the newsvendor equations for the Clark–Scarf system to serial systems with a fixed batch size per stage. It also contributes to a better understanding of how these results are derived. A key step in the analysis of this paper is constituted by the introduction of the so-called shortfalls.

Notice that newsvendor equations also play a key role in a recent research stream on bounds and approximations for optimal basestock/reorder levels and optimal costs in multi-echelon systems; see Shang and Song [8] and the references therein. The above research on newsvendor equations for the optimal basestock/reorder levels themselves is related and might support the derivation of alternative bounds and approximations.

In this paper, we limit ourselves to the continuous demand case, because the newsvendor equations are much cleaner for continuous demand than for discrete demand. The discrete demand case has been analyzed in [6]. In that case one obtains newsvendor inequalities, as in the standard single-stage newsvendor problem. Our analysis is based on results of Chen [2,3]. Those results have been derived for the discrete demand case, but, by considering a continuous demand distribution as a limiting distribution of a series of discrete demand distributions, it is easily seen that those results also hold for continuous demand.

The organization of this paper is as follows. We complete the model description in Section 2, and the whole analysis and main results are presented in Section 3.

2. Model

The main assumptions of our model have been described above. In this section, we give additional assumptions at a more detailed level and we introduce additional notation.

We assume the following sequence of events per period: (i) inventory levels at all stages are observed and the current period's ordering decisions are made (at the beginning of the period); (ii) orders arrive following their respective leadtimes (at the beginning of the period); (iii) demand occurs during the period; (iv) holding and penalty costs are assessed (at the end of the period).

We follow essentially the same notation and assumptions as Chen [2]. However, we assume continuous demand, as argued in the introduction, and consider a periodic review setting. As indicated by Chen, his results also hold for the periodic review case.

In addition to the notation that has been introduced already, we define:

t	index for periods, $t \in \mathbb{N}_0 := \mathbb{N} \cup \{0\}$;
L_i	leadtime from stage $i+1$ to stage i, $L_1 \in \mathbb{N}_0$ and $L_i \in \mathbb{N}$ for $i = 2, \ldots, N$;
l_i	total leadtime from the external supplier to stage i, $l_i = \sum_{j=i}^{N} L_j$ for $i = 1, \ldots, N$;
h_i	echelon holding cost per unit per period at stage i, $h_i > 0$ for $i = 1, \ldots, N$;
H_i	installation holding cost per unit per period at stage i, $H_i = \sum_{j=i}^{N} h_j$ for $i = 1, \ldots, N$;
p	penalty cost per backlogged unit per period, $p > 0$;
$D(t)$	demand in period t, which is continuously distributed on $[0, \infty)$ with $\mathsf{P}\{D(t) = 0\} = 0$, i.e., there is no positive probability mass in 0;
F	cumulative distribution function of one-period demand;
μ	expected one-period demand, $\mathbf{E}[D(t)] = \mu$ for all t, $\mu > 0$;
$D_i(t)$	demand during the periods $t + l_{i+1}, \ldots, t + l_i$;
$D_i^-(t)$	demand during the periods $t + l_{i+1}, \ldots, t + l_i - 1$;
$B(t)$	backorder level at stage 1 at the *end* of period t;
$IL_i(t)$	echelon inventory of stage i at the *end* of period t, i.e., on-hand inventory at stage i plus inventories in transit to or on-hand at stages $1, \ldots, i-1$ minus backorders at stage 1;
$IL_i^-(t)$	echelon inventory of stage i at the *beginning* of period t just after the receipt of the incoming order, but before the demand, $IL_i(t) = IL_i^-(t) - D(t)$;
$IP_i(t)$	echelon inventory position at stage i at the *beginning* of period t just after ordering, but before the demand, i.e., $IL_i^-(t)$ $+$inventories in transit to stage i.

When the period index t in variables $D_i(t), D_i^-(t), B(t), IL_i(t), IL_i^-(t)$ and $IP_i(t)$ is suppressed, the notation represents the corresponding steady state variables. While D_i^- denotes demand over an interval of L_i periods, D_i stands for demand over an interval of $L_i + 1$ periods.

We assume linear inventory holding and penalty costs because the newsvendor equations are only obtained under this cost structure. However, the new average costs formulas in Section 3.2 can be extended to general functions for the so-called echelon costs.

3. Analysis

In this section, we first review some important results from [2] in Section 3.1. After that, the new costs formulas and newsvendor equations are derived in Sections 3.2 and 3.3, respectively.

3.1. Preliminaries

The costs in any period may be expressed via echelon holding cost parameters. At the end of a period t, the expected holding and penalty costs are equal to

$$\sum_{i=1}^{N} h_i IL_i(t) + (p + H_1)B(t). \tag{1}$$

The costs $h_i IL_i(t)$ are the *costs attached to echelon* i, $2 \le i \le N$, and the costs $h_1 IL_1(t) + (p + H_1)B(t)$ are the *costs attached to echelon* 1.