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A general model for the undesirable single facility location problem☆

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Abstract

In this paper, a finite set in which an optimal solution for a general Euclidean problem of locating an undesirable facility in a polygonal region, is determined and can be found in polynomial time. The general problem we propose leads us, among others, to several well-known problems such as the *maxisum*, *maximin*, *anticentdian* or *r-anticentrum* problem.

Keywords: Location; Undesirable facilities; Obnoxious facilities; Maximin; Maxisum

1. Introduction

The facilities may be categorized in a general fashion as being either desirable, in which case the facilities should be closer to the users, or undesirable when they should be far away. Many facilities provide benefits or services to their users while having an adverse effect on the people near by. Such facilities include chemical plants, garbage disposal sites, nuclear power stations, plants for treatment of residual waters, airports, etc. The growing interest in location modelling for undesirable facilities may be attributed to our growing concerns over the environment. In fact, any type of modern facility will have some detrimental effects on the quality life, as a result of human perceptions evidenced by the physical effects of different forms of pollution such as air, water and noise pollution.

In general, the objective is to locate the facility as far away as possible from an identified set of population centers; the *maxisum* criterion attempts to maximize the sum of the distances from the undesirable (obnoxious) facility to the population centers and the optimal facility location will always be on the boundary

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of the feasible region. It is more appropriate when an aggregate measure of quality is desired and each population center has a measurable contribution in the objective function. The *maximin* criterion attempts to maximize the minimum level of quality among the population centers and it guarantees that the optimal location will not be too close to any population center.

Using the weighted Voronoi diagram in the plane, Melachrinoudis and Smith [6] have developed a $O(nm^2)$ algorithm for the Euclidean weighted maximin problem in a polygonal region with m edges, being n the number of existing facilities. Romero et al. [12] models semi-obnoxious facility location taken into account both the environmental impact and the transportation costs and they have proposed a solution method based on the well-known big square–small square (BSSS). Brimberg and Juel [1] have described a trajectory method for constructing an efficient frontier of points for a bicriteria model for locating a semi-desirable facility in the plane (the first criterion is used to measure the transportation costs and the second one estimates the social or environmental cost) by varying the relative weights, where the weighted sum of the two criteria is minimized. Welch and Salhi [14] have proposed three heuristics to solve the maximin formulation for siting p facilities on a network considering a pollution dispersion equation and facility interaction. Plastria and Carrizosa [10] have proposed a bicriteria model seeking the lowest affection of population at the highest level of protection. They have developed fast polynomial algorithms to construct the complete trade-off curve between both objectives together with corresponding efficient solutions. Fernández et al. [5] have presented a model that minimizes the global repulsion of the inhabitants of the region while taking into account environmental concerns which make some areas suitable for the location of the facility. Rodríguez-Chía et al. [11] have described the ordered Weber problem which is a generalization of the Weber problem. They study the minimum of this function and obtain some properties that they use for suggest an algorithm and other applications.

In this paper, we study the weighted problem of locating an undesirable facility in a polygonal region that includes several existing population centers. This problem was proposed by Saameño [13] and a characterization of the solution set for the unweighted model can be seen in Muñoz and Saameño [9].

The present paper is organized as follows: Section 2 states the proposed model and describes its properties. Section 3 contains the results that identify the finite dominating set of the polygonal region. Section 4 contains some results to reduce the candidate point set in the search process for the optimal solution. Section 5 describes the proposed algorithm and some computational results. Finally, Section 6 contains some remarks and conclusions.

2. Problem statement and properties

The problem is defined as follows: let S be a closed polygonal region of $\mathbb{R}^2$ and $\{P_1, P_2, \ldots, P_m\}$ a finite subset of points of the plane $\mathbb{R}^2$, corresponding to the population centers. Let $\vec{k} = (k_1, k_2, \ldots, k_m)$ be a given vector of m non-negative components and let w_i be the positive weight associated to P_i, that is, the importance given to the existing facility P_i.

Let $\|\cdot\|$ be the Euclidean norm and, for each $x \in S$, let σ be the permutation of the set $\mathscr{M} = \{1, 2, \ldots, m\}$ such that

$$w_{\sigma(1)}\|x - P_{\sigma(1)}\| \leqslant w_{\sigma(2)}\|x - P_{\sigma(2)}\| \leqslant \cdots \leqslant w_{\sigma(m)}\|x - P_{\sigma(m)}\|.$$

If we consider the objective function defined as

$$f(x) = \sum_{i=1}^{m} k_i w_{\sigma(i)}\|x - P_{\sigma(i)}\|,$$

then our problem $(\mathscr{P})$ is formulated as

$$\max_{x\in S} f(x) = \max_{x\in S} \sum_{i=1}^{m} k_i w_{\sigma(i)}\|x - P_{\sigma(i)}\|.$$

Note that this problem includes well-known problems:

- If $k_i = 1$, $\forall i \in \mathscr{M}$, we obtain the *maxisum* problem.
- If $k_1 = 1$ and $k_i = 0$ for $i > 1$ then we obtain the *maximin* location problem.
- If $k_r = 1$ and $k_i = 0$ for $i \neq r$ then the problem is to maximize the rth closest distance between the undesirable facility and the affected centers. It is an extension of the maximin location problem called the rth quantile location problem.