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Cost optimization in the $(S-1, S)$ lost sales inventory model with multiple demand classes

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Abstract

For the $(S-1, S)$ lost sales inventory model with multiple demand classes that have different lost sales penalty cost parameters, three accurate and efficient heuristic algorithms are presented that, at a given base stock level, aim to find optimal values for the critical levels, i.e., values that minimize inventory holding and penalty cost.

Keywords: Inventory; Multiple demand classes; Customer differentiation; Rationing; Lost sales

1. Introduction

We study the $(S-1, S)$ lost sales inventory model with multiple demand classes. A class-dependent penalty is incurred if a demand is not fulfilled from stock. The penalty represents contractual penalty cost, (additional) cost of emergency transportation and/or loss of goodwill of a customer. We aim to minimize the total inventory holding and penalty cost, and we distinguish between different classes by introducing critical levels. A demand from a certain class is only fulfilled if the physical stock is above the critical level for this class.

The problem of multiple demand classes has been introduced by Veinott [11], who also introduced the concept of critical level policies. After that, several variants have been studied, and two streams of research may be distinguished. In the first stream, the optimality of critical level policies for single-item models with multiple demand classes has been derived. Topkis [10] did this for a periodic-review model with generally distributed demand and zero lead time, in which case the optimal critical levels depend on the remaining time in a period. Ha [4] derived the optimality of critical level policies for a continuous-review model with Poisson demand processes, a single exponential server for replenishments, and lost sales; this work has been extended by de Véricourt et al. [12] to the backordering case. In these cases, the base stock levels and critical levels are time-independent. In the

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second stream, evaluation and optimization within a given class of policies is considered. Dekker et al. [2] derived exact and heuristic procedures for the generation of an optimal critical level policy for a continuous-review model with multiple customer classes, Poisson demands, ample supply, and lost sales. For the case with two classes, Melchiors et al. [9] extended this work for fixed quantity ordering. Deshpande et al. [3] considered a similar model but with backordering of unsatisfied demand. The order in which backordered demands are satisfied leads to additional complications. Further references are found in the above papers.

In this paper, we study a single-item, continuous-review model with multiple customer classes, Poisson demand processes, lost sales, and ample supply. The ample supply represents that the supplier can always deliver within a given replenishment lead time. We limit ourselves to the class of critical level policies with time- and state-independent critical and base stock levels. Critical levels are easy to explain in practice, and the results on optimal policies in the first stream of research suggest that this class is at least close-to-optimal. Under the given policy structure, our problem is to optimize $|J|$ critical levels and one base stock level simultaneously, where J is the set of demand classes. By a projection of the $(|J|+1)$-dimensional total cost function on the dimension of the base stock level and the definition of appropriate convex lower and upper bound functions, the full $(|J|+1)$-dimensional optimization problem may be reduced to solving a series of $|J|$-dimensional subproblems; see [2]. Each subproblem concerns the optimization of the $|J|$ critical levels at a given base stock level. By this reduction and applying enumeration for the subproblems, an exact solution method is obtained for the full problem. Enumeration, however, is expensive from a computational point of view, especially when the number of demand classes is larger than two. Therefore, there is a clear need for efficient heuristics for the $|J|$-dimensional subproblems.

Our main contribution is that three heuristic algorithms are presented for the $|J|$-dimensional subproblems. An extensive computational experiment is performed, and, surprisingly, all three algorithms always end up in an optimal solution. The size and settings of the experiment make us believe that the algorithms are either very good (i.e., find an optimal solution in many cases) or exact (i.e., always find an optimal solution). We conjecture the latter. For practical application, even if our conjecture does not hold, we still have obtained an important result. Computation times of the three heuristics are small, and far less than of explicit enumeration. So, the three heuristics are accurate and efficient. The three heuristics directly lead to three accurate and efficient heuristics for the full $(|J|+1)$-dimensional problem, and that may be key to obtain efficient solution methods for real-life, multi-item spare parts problems with multiple demand classes and aggregate fill rate or availability constraints. These problems may be solved by a Dantzig–Wolfe decomposition framework, under which multiple instances of a single-item problem with inventory holding and penalty cost have to be solved as column generation subproblems. Our heuristics may be used for these column generation subproblems in all but the last iteration, followed by an exact algorithm in the last iteration. That avoids using an exact method in all iterations and decreases the order of computation time considerably without losing the property that the Dantzig–Wolfe method is exact; see the companion paper [8]. Another contribution is that we derive basic monotonicity properties for the fill rates of all demand classes and average costs as a function of critical levels.

The paper is organized as follows. In Section 2, the model is described, followed by the derivation of the monotonicity properties in Section 3. The exact method for the full $(|J|+1)$-dimensional problem is briefly described in Section 4. In Section 5, we present our three efficient heuristic algorithms for the $|J|$-dimensional subproblem, and we show that these algorithms always lead to an optimal solution in an extensive computational experiment. In Section 6, we apply these algorithms in the original full problem and we compare them to a heuristic of [2].

2. Model

Consider an item that is demanded by multiple demand classes. Let J be the set of demand classes ($|J| \geqslant 1$). For each class $j \in J$, demands occur according to a Poisson process with rate m_j (>0). If an item is not delivered to class j upon request, the demand is lost and a penalty cost p_j (>0) is to be paid. Classes are numbered $1, 2, \ldots, |J|$ and such that $p_1 \geqslant p_2 \geqslant \cdots \geqslant p_{|J|}$.