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Dynamic lot-sizing model with demand time windows and speculative cost structure

Hark-Chin Hwang[a,*,1], Wikrom Jaruphongsa[b]

[a] *Department of Industrial Engineering, Chosun University, 375 Seosuk-Dong, Dong-Gu, Gwangju 501-759, South Korea*
[b] *Department of Industrial and Systems Engineering, National University of Singapore, 10 Kent Ridge Crescent, Singapore 119260, Singapore*

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Abstract

We consider a deterministic lot-sizing problem with demand time windows, where speculative motive is allowed. Utilizing an untraditional decomposition principle, we provide an optimal algorithm that runs in $O(nT^3)$ time, where n is the number of demands and T is the length of the planning horizon.

Keywords: Dynamic lot-sizing model; Demand time window; Speculative motive; Dynamic programming

1. Introduction

For demands of a single item, the classical dynamic lot-sizing model focuses on decisions about when and in what quantity to produce or order to minimize the total ordering and inventory-holding costs over the planning horizon T [10]. The costs being considered are linear production (procurement) cost for each unit produced, fixed setup (ordering) costs incurred whenever the item is produced or ordered, and linear inventory-holding costs [3,13]. Also, to deal with shortages, linear backlogging cost is considered [2,9]. In the *non-speculative* cost structure, the production cost plus inventory-holding cost in period t is not cheaper than the production cost in period $t + 1$ and the production cost plus backlogging cost in period t is not less than the production cost in period $t - 1$. For the general cost structure with no such restrictions on costs, we say that *speculative motive* is allowed as opposed to the non-speculative cost structure. Zangwill [12] further generalized the problem by considering a concave cost structure. Three important papers [1,4,10] improved the time complexity for obtaining an optimal solution from $O(T^2)$ to $O(T \log T)$ for general problems and to $O(T)$ for problems with a non-speculative cost structure.

For decades, the study of the dynamic lot-sizing model has mainly dealt with the situation where the

* Corresponding author.
E-mail address: hchwang@chosun.ac.kr (H.-C. Hwang).
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requirements are aggregated by period and thus each demand is defined per period. However, as the relationship between supplier and customer gets closer and collaboration is a key competitive strategy in supply chains, demands are often established by long-term agreement in bulk and in multi-periods. For these reasons, the due dates are given by intervals of periods, and no costs of inventory holding and backlogging are accrued whenever demands are satisfied during the intervals. This interval of grace periods is called a *time window*. In this case, demands are aggregated by time windows rather than by periods. Lee et al. [8] were the first to study the dynamic lot-sizing model with demand time windows. Under the non-speculative cost structure, they provided two optimal algorithms for the no backlogging case and for the backlogging case with computational complexities of $O(T^2)$ and $O(T^3)$, respectively.

Recently, Jaruphongsa et al. [6,7] considered the dynamic lot-sizing model with time windows for a three-stage supply chain. Similar to Lee et al. [8], their optimal algorithms also rely heavily on the non-speculative cost assumption. To the best knowledge of the authors, no result has been published for the dynamic lot-sizing model with time windows that allows a speculative cost structure. One might wonder whether or not the problem is NP-Hard. Hwang and Jaruphongsa [5] considered a single item dynamic lot-sizing problem with demand time windows under a concave cost structure. They provided an optimal algorithm for a special case where the time window of a demand does not superimpose on time windows of other demands. In this paper, we generalize the Lee et al. [8] model by removing the non-speculative cost assumption. For this model with backlogging allowed, we propose an optimal algorithm with computational complexity of $O(nT^3)$, where n is the number of demands. Namely, the problem is not NP-Hard. It would be an interesting question whether another optimal algorithm with less computing time than $O(nT^3)$ could be designed even in the special case that backlogging is not allowed. For problems with more general cost structures (e.g. concave cost structures), it is still an open question whether or not there exists an optimal procedure that runs in polynomial time.

In the next section, we present a mathematical model of the problem. We review useful optimality properties provided in [5,8] in Section 3. The efficient algorithm based on a backward dynamic programming procedure is presented in Section 4.

2. The model

Suppose that we have n demands to be satisfied over the planning horizon T. Note that n is $O(T^2)$ in general. We define the following parameters and decision variables for our model.

Parameters:

- d_i denotes the required quantity for demand i for $i = 1, \dots, n$.
- $[E_i, L_i]$ denotes the time window of demand i for $i = 1, \dots, n$ during which no costs of inventory holding and backlogging are incurred. E_i and L_i are called the *earliest* and *latest* due dates of demand i, respectively.
- K_t denotes the fixed cost of production in period t.
- p_t denotes the unit production cost in period t. We let $p_0 = p_{T+1} = \infty$.
- h_t denotes the unit holding cost in period t. We let $h_0 = \infty$.
- g_t denotes the unit backlogging cost in period t.

Decision variables:

- y_{it} denotes the amount dispatched in period t for demand i for $i = 1, \dots, n$ and $t = 1, \dots, T$.
- x_t denotes the amount replenished in period t for $t = 1, \dots, T$.
- I_t^+ denotes the inventory level in period t for $t = 1, \dots, T$.
- I_t^- denotes the backlogging level in period t for $t = 1, \dots, T$.

The mathematical formulation of the problem is given by

$$
\begin{aligned}
&\text{minimize} && \sum_{t=1}^{T} (K_t \delta(x_t) + p_t x_t + h_t I_t^+ + g_t I_t^-), \\
&\text{subject to} \\
& x_t + (I_{t-1}^+ - I_{t-1}^-) - \sum_{i=1}^{n} y_{it} = (I_t^+ - I_t^-), && t=1,\dots,T, \\
& \sum_{t \in [E_i, L_i]} y_{it} = d_i, && i=1,\dots,n, \\
& y_{it} \geqslant 0, && i=1,\dots,n,\ t \in [E_i, L_i], \\
& y_{it} = 0, && i=1,\dots,n,\ t \notin [E_i, L_i], \\
& x_t \geqslant 0,\ I_t^+ \geqslant 0,\ I_t^- \geqslant 0, && t=1,\dots,T, \\
& I_0^+ = I_0^- = I_T^+ = I_T^- = 0,
\end{aligned}
$$