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The competitive salesman problem on a network: a worst-case approach

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Abstract

We provide a complexity analysis of the problem of optimal routing of a server on a transportation network in the presence of a competing server. The server that reaches a node first gets the profit from the node. The objective is to maximize the worst-case profit.

Keywords: Polynomial algorithm; NP-hardness; Traveling salesman

1. Introduction

A large amount of Operations Research literature is devoted to optimization models of competitive situations. For example, there is an extensive literature on competitive location models (see, e.g., [3,4] and the references therein), where firms compete for customers located on a transportation network by trying to attract them with convenient locations of facilities.

Another type of a competitive situation is where firms dispatch servers (e.g., salesmen or repairmen) to offer products or services on-site. Here, the assumption is that the service or products are offered to customers without preliminary arrangement or phone advertisement, because a customer is likelier to accept the service or the products in a "face-to-face" situation rather than by phone, or because the product needs a demonstration. If there are two or more firms offering similar service, then a customer would accept the service from the firm whose representative (server) reaches the customer first. Then, if the competing firms dispatch the servers simultaneously, their profits depend on the tours (sequences of visiting the customers) that they choose for their servers.

To our knowledge, this type of competitive routing has not been analyzed in the literature. The paper represents a first step in this direction. We analyze the complexity of the problem of finding an optimal

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sequence of visiting customers located at nodes of a transportation network by a single server (called the *main* server) in the presence of a competing server (called the *competitor*), assuming that both servers start travel simultaneously from their respective home locations and travel with the same speed. The objective is to maximize the worst-case profit of the server. We assume that the server who reaches a customer first gets the profit from that customer, and the other server gets nothing. If both servers reach a customer simultaneously, we consider two possibilities: (a) The main server gets the profit from the customer (the case of a *dominant main server*); (b) The servers split equally the profit from the customer (the case of *equal* servers). We distinguish between the case of equal home locations of the servers and the case of different home locations. With respect to the information structure of the decision, we distinguish between the "static" case where a tour is chosen a priori and cannot be modified on the way based on new information about the actions of the competitor, and the "dynamic" case where a server can use information about actions of the competitor to modify its tour during the travel.

Our main results are as follows. All considered static problems are strongly NP-hard if the home locations of the servers are different. If the home locations are the same, the static problem can be solved in $O(m + n \log n)$ time for the case of a dominant main server (m is the number of edges, and n is the number of nodes of the network), but (somewhat surprisingly) it is NP-hard for the case of equal servers. Moreover, for the case of equal servers, it is NP-hard even to find the worst-case profit of the main server given its tour. The dynamic problem is NP-hard in all versions on general networks; in the case of a dominant main server, it is NP-hard even on a star network with equal home locations.

Section 2 presents notation and definitions, Sections 3 and 4 present the results for the static problem, Section 5 presents the results for the dynamic problem. Directions for future research are in Section 6.

Information and references on routing problems without competition can be found in [6]. Some scheduling problems with competition have been considered in [2].

2. Preliminaries

Let $G=(V, E)$ be an undirected network with V the set of nodes and E the set of edges, $|V|=n$, $|E|=m$. Each edge $e \in E$ has a positive integer length l_e. There are two servers, Servers 1 and 2, initially located at their respective home locations. Each server has to visit all nodes of the network and return back to its home location. Both servers start traveling simultaneously and travel with the unit speed. Server 1 will also be called the *main server*, and Server 2 will also be called *the competitor*. A nonnegative integer weight w_v is associated with each node $v \in V$ and represents the possible profit at the node; the server who reaches the node first gets the profit (the server who reaches the node second gets nothing). If there is a conflict (both servers reach a node simultaneously), we consider two cases:

Case 1: *Dominant main server*. Server 1 gets all the profit at the node, Server 2 gets nothing.

Case 2: *Equal servers*. The profit at the node is divided equally between the servers.

We consider the problem of finding a tour for the main server that maximizes its worst-case profit, assuming that the tour is chosen in advance, before the servers start travel, and cannot be modified during the travel (that is, an information about actions of the competitor cannot be used to modify the tour).

A more formal statement of the problem is as follows. Let T_1 (T_2) be the set of possible tours for Server 1 (Server 2); let $A(y_1, y_2)$ be the profit of Server 1 if it uses tour y_1 and Server 2 uses tour y_2. Then, if Server 1 uses a tour $y_1 \in T_1$, its worst-case profit is

$$Z(y_1) = \min_{y_2 \in T_2} A(y_1, y_2) \tag{1}$$

and the problem can be stated as

Problem TOUR. Maximize $\{Z(y_1) \mid y_1 \in T_1\}$.

The problem of finding the value $Z(y_1)$ for a specific $y_1 \in T_1$ will be called **Problem ANTITOUR($\mathbf{y_1}$)**, and the corresponding minimizing tour for the competitor in (1) will be called an *antitour* for y_1. In the case of a dominant main server, Problem ANTITOUR(y_1) is trivial, regardless of whether the home locations are the same or different: the competitor should go to the first node of the tour y_1 that it can reach before the main server, and then to imitate the remainder of the