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On the discrete lot-sizing and scheduling problem with sequence-dependent changeover times

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ABSTRACT

We consider the discrete lot-sizing and scheduling problem with sequence-dependent changeover costs and times and propose solving it as a mixed-integer program using a commercial solver. Our approach is based on the extension of an existing tight formulation for the case without changeover times. Computational results confirm the benefits of the proposed solution procedure.

1. Introduction

A wide variety of models for production planning and inventory management have been investigated in operations research. Among them, capacitated lot-sizing models aim at determining the optimal timing and level of production complying with given capacity restrictions and such that demand for all products is satisfied without backlogging. Recent overviews on the lot-sizing literature can be found among others in [2,4].

In the present paper, the discrete lot-sizing and scheduling problem (DLSP) is considered. The DLSP relies on several basic assumptions (see e.g. [3]):

- Demand for products is deterministic and time-varying.
- The production plan is established for a finite time horizon subdivided into several discrete periods.
- At most one item can be produced per period ("small bucket" model) and the facility processes either one product at full capacity or is completely idle ("all-or-nothing assumption").
- Costs to be minimized are the inventory holding costs and the changeover costs.

Here the single level single machine variant of this problem is studied: all items to be produced are end items and share the same constrained resource. In the DLSP, it is assumed that there is a changeover between two production runs for different items, resulting in a changeover cost and/or a changeover time. Changeover costs and times can depend either on the next item only (sequence-independent case) or on the sequence of items (sequence-dependent case). Significant changeover times which consume scarce production capacity tend to further complicate the problem. We consider here the most difficult variant: the DLSP with sequence-dependent changeover costs and times (denoted DLSPSD in what follows).

The DLSP has received much attention in the literature. Complexity results for this problem can be found in [8]. They show that the single machine multi-product case without setup times is NP-hard and that the problem of finding a feasible solution in the presence of sequence-independent setup times is NP-complete. We deal here with the extension of this problem to the case of sequence-dependent changeover costs and times. The DLSPSD under study in the present paper is thus NP-hard.

We now discuss in more detail specific contributions on the DLSPSD. [9] reformulates the problem as a travelling salesman problem with time windows and uses a dynamic-programming-based algorithm to solve it. [5] shows the equivalence between the DLSPSD and the batch sequencing problem (BSP) and uses a specific branch and bound type algorithm for solving the BSP to optimality. In both papers, the mixed-integer programming formulation proposed for the problem is weak and does not provide lower bounds good enough to solve the problem using a commercial solver (see the results in Section 3.2). However, as pointed out by [7], there is now a good knowledge about the "right" way to formulate many simple production planning submodels as mixed-integer programs and, thanks to it, many practical production planning problems can be (approximately) solved using commercial solvers. To the best of our knowledge,

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Table 1
Mixed-integer formulation DLSPSD2.

$\min \sum_{i=1}^{N}\sum_{t=1}^{T} h_i I_{it} + \sum_{i=0}^{N}\sum_{j=0}^{N}\sum_{t=1}^{T} c_{ij} w_{ijt}$		(3)
$I_{it} = I_{i,t-1} + P_{it} y_{it} - d_{it}$	$\forall i = 1 \ldots N,\ \forall t = 1 \ldots T$	(4)
$y_{i,t-1} = \sum_{j=0}^{N} w_{ijt}$	$\forall i = 0 \ldots N,\ \forall t = 1 \ldots T$	(5)
$y_{jt} = \sum_{i=0 \ldots N \text{ st } t-T_{ij}>0} w_{ij,t-T_{ij}}$	$\forall j = 0 \ldots N,\ \forall t = 1 \ldots T$	(6)
$\sum_{i=0}^{N} y_{it} + v_t = 1$	$\forall t = 0 \ldots T$	(7)
$I_{it} \geq 0$	$\forall i = 1 \ldots N,\ \forall t = 1 \ldots T$	(8)
$y_{it} \in \{0, 1\}$	$\forall i = 0 \ldots N,\ \forall t = 0 \ldots T$	(9)
$w_{ijt} \in [0, 1]$	$\forall i = 0 \ldots N,\ \forall j = 0 \ldots N, \forall t = 1 \ldots T$	(10)
$v_t \in [0, 1]$	$\forall t = 0 \ldots T$	(11)

these results have not yet been exploited to solve the DLSPSD. In the present paper, we attempt to close this gap by proposing a new tight formulation for this specific variant of the problem.

The purpose of this paper is thus to introduce a strengthened formulation for the DLSP with sequence-dependent changeover costs and times. This formulation is an extension of the formulation proposed by [11] for the DLSP with sequence-dependent changeover costs and zero changeover times. Thanks to this strengthened formulation, the lower bounds provided by the linear relaxation of the problem are significantly better, enabling a branch and bound type procedure to solve the problem more efficiently.

The paper is organized as follows. In Section 2, we present the proposed tight formulation for the DLSPSD. In Section 3, we discuss the results of some computational experiments carried out to evaluate it.

2. A tight formulation for the DLSPSD

We present a tight formulation for the DLSP with sequence-dependent changeover costs and times. This formulation is an extension of the formulation proposed by [11] for the DLSP with sequence-dependent changeover costs and zero changeover times. In what follows, we denote it as the DSLPSD2 formulation whereas we denote as DLSPSD1 the formulation proposed by [9].

2.1. Basic formulation

We wish to optimize the production schedule for a set of N items over an horizon featuring T planning periods. A period is indexed by $t = 1, \ldots, T$, an item by $i = 0, \ldots, N$. We agree to use item $i = 0$ to represent idle periods.

We use the following notation:

- d_{it}: demand (in units) for item i in period t.
- P_{it}: production capacity (in units per period) for item i in period t.
- h_i: holding costs per unit and period for item i.
- c_{ij}: changeover costs from item i to item j.
- T_{ij}: changeover time from item i to item j. T_{ij} is assumed to be an integer number of planning periods.

Decision variables are defined as follows:

- I_{it}: inventory level corresponding to item i at the end of period t.
- y_{it}: setup variables. y_{it} equals 1 if the resource is setup for production of item i in period t, and 0 otherwise.
- w_{ijt}: changeover cost variables. If $T_{ij} > 0$, w_{ijt} equals 1 during the first period of a changeover from item i to item j, and 0 otherwise. If $T_{ij} = 0$, w_{ijt} equals 1 in the first period of production of j, and 0 otherwise.
- v_t: changeover time variables. v_t equals 1 during each period in which a changeover between two items occurs, and 0 otherwise.

In the mixed-integer formulation proposed in Table 1, the objective (3) minimizes the sum of inventory holding costs and changeover costs. Note that, in the DLSPSD2 formulation, variables w_{iit} are introduced: $w_{iit} = 1$ means that the resource is setup for item i both in period $t-1$ and in period t, i.e. that a production run for item i takes place over periods $t-1$ and t.

Constraints (4) express the inventory balance. Together with constraints (8), they ensure that demand for each item is fulfilled without backlogging.

Equalities (5) and (6) link the setup variables with the changeover cost variables. (5) guarantee that item i can be produced in period $t-1$ if and only if a changeover from i to another item j (possibly $j = i$) takes place at the beginning of period t. Similarly, (6) guarantee that item j can be produced in period t if and only if a changeover from another item i (possibly $i = j$) to item j begins early enough (i.e. in period $t - T_{ij}$) to be finished at the beginning of period t.

(7) ensure that in each period, the resource either produces a single product at full capacity, or is idle (i.e $y_{0t} = 1$), or is in transition between two items (i.e. $v_t = 1$).

The binary character of the setup variables is represented by (9). (10) and (11) state the non-negativity of the changeover variables: observe, as pointed out by [1], that thanks to constraints (5)–(7) and (9), there is no need to define variables w_{ijt} and v_t as binary variables.

2.2. Valid inequalities

As shown in [11] for the case without changeover times, the DLSPSD2 formulation can be further strengthened through a family of valid inequalities adapted from the ones developed by [10]. We investigate here an extension of this idea to the case of positive changeover times and propose a family of valid inequalities for the problem (3)–(11).

This can be done using the assumption of Wagner–Whitin costs, constant capacity and no backlogging. In this case, demands and production capacity can be normalized without loss of generality: $d_{it} \in \{0, 1\}$ and $P_{it} = 1$. We first introduce some additional notation:

- $D_{i,t,\tau}$: cumulated demand for item i in the interval $\{t, \ldots, \tau\}$. Demand on item i is binary so that $D_{i,t,\tau}$ is equal to the number of positive demand periods for i in $\{t, \ldots, \tau\}$.
- $S_{i,q}$: qth positive demand period for item i. Note that $S_{i,D_{i,1,t}+q}$ denotes the qth period with positive demand for item i after period t.

We also introduce the start-up variables z_{it} defined as follows: z_{it} equals 1 if the production of a new lot of item i starts at the