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E-Bayesian Estimation and Hierarchical Bayesian Estimation for Estate Probability in Engineering

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Abstract

Dr.Han Ming gives the definition of E-Bayesian estimation of estate probability and the formulas of E-Bayesian estimation, engineering forecast model and its applications in security investment at 2005. He gives hierarchical Bayesian estimation of estate probability and guess that the E-Bayesian estimation asymptotic equal to hierarchical Bayesian estimation of estate probability at 2006. In this paper we give the proof of that the limits of E-Bayesian estimation and hierarchical Bayesian estimation for estate probability are both zero and the hierarchical Bayesian estimation of estate probability which has no observation data is less than that of E-Bayesian estimation, and guess that the hierarchical Bayesian estimation of estate probability which has observation data is also less than that of E-Bayesian estimation.

Keywords: Parameter estimation; estate probability; E-Bayesian estimation; hierarchical Bayesian estimation, engineering

1. Introduction

In 2005, Dr. Han Ming gives the definition of E-Bayesian estimation of estate probability、E-Bayesian estimation formula, forecast model and its application in securities investments[1].In 2006, he gives hierarchical Bayesian estimation of state probability asymptotic equal to E-Bayesian estimation[2].the paper is only given the

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engineering numerical calculation examples, but isn't given the mathematical proof. In this paper we will give the proof of that the limits of E-Bayesian estimation and hierarchical Bayesian estimation for state probability are both zero .As in the table 2 and table 3 of the paper [2], the numerical calculation of hierarchical Bayesian estimation is error, so Han Ming fail to draw on the size comparative between estate probability's hierarchical Bayesian estimation value and E-Bayesian estimation value. We can see that hierarchical Bayesian estimation value of state probability is less than that of E-Bayesian estimation value from correct simulation of hierarchical Bayesian estimation .For no observation data, the hierarchical Bayesian estimation of estate probability is less than that of E-Bayesian estimation. And for observation data, we can guess that the engineering hierarchical Bayesian estimation of estate probability is also less than that of E-Bayesian estimation by engineering numerical examples.

2. E-Bayesian estimation and hierarchical Bayesian estimation of state probability

Suppose that $x_1, x_2, \cdots, x_n$ are sample observed values from the same population. Then we divide the n-observations into k groups $(1 < k < n)$ at a certain distance. Accordingly, the forecast objects are divided into k states, let $E_i (i = 1,2,\cdots,k)$ be the ith state, there are $r_i (0 \le r_i < n, i = 1,2,\cdots,k)$ observed values in state E_i .Let $p_i (i = 1,2,\cdots,k)$ be the probability of forecast objects falling into state $E_i (i = 1,2,\cdots,k)$, we shall call it estate probability. Suppose the event whether the forecast objects fall into the state E_i is independent. Then the forecast of securities price can be approximated as n-Bernoulli trials problem, so it can be described as binomial distribution. Let X be the times of forecast objects falling into the state E_i , and its distribution law is

$$P(X = r_i) = C_n^{r_i} p_i^{r_i} (1 - p_i)^{n - r_i} \quad , r_i = 0,1,2,\cdots,n$$

Where $0 < p_i < 1$, p_i is the estate probability.

If the prior distribution of estate probability p_i is Bata distribution, and its probability density function is

$$\pi(p_i \mid a,b) = \frac{p_i^{a-1}(1-p_i)^{b-1}}{B(a,b)}$$

Where $0 < p_i < 1, a > 0, b > 0$, a and b are hyper-parameters. $B(a,b) = \int_0^1 t^{a-1}(1-t)^{b-1}\mathrm{d}t$ is Bata distribution.

When $0 < b < 1, a > 1$, $\pi(p_i \mid a,b)$ is increasing in p_i , and meets the requirements of hierarchical prior distribution decreasing function's construction. Prior distribution with thinner tail would make worse robustness of Bayesian distribution. So when $0 < b < 1$, a should not be too large. Let the upper bound of b be c ($c > 1$, and c is a constant). The value range of Hyper-parameters a and b is $D = \{(a,b) \mid 1 < a < c, 0 < b < 1\}$. Suppose the prior distribution of a is uniform distribution in $(1,c)$, and the prior distribution of b is uniform distribution in $(0,1)$, when a and b are independent, hierarchical prior density distribution of θ is given by

$$\pi(p_i) = \int_1^c \int_0^1 \pi(p_i \mid a,b)\pi(a,b)\mathrm{d}b\mathrm{d}a = \frac{1}{(c-1)} \int_1^c \int_0^1 \frac{p_i^{a-1}(1-p_i)^{b-1}}{B(a,b)}\mathrm{d}b\mathrm{d}a , \quad 0 < p_i < 1 .$$

For simplicity, the following only consider the case when $a = 1$ .now the prior distribution of estate probability p_i is

$$\pi(p_i \mid b) = b(1-p_i)^{b-1} \tag{1}$$