Journal of the Korean Statistical Society 45 (2016) 14–24

Contents lists available at ScienceDirect

Journal of the Korean Statistical Society

journal homepage: www.elsevier.com/locate/jkss

A production–inventory system with a Markovian service queue and lost sales

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ARTICLE INFO

Article history:
Received 20 March 2015
Accepted 12 May 2015
Available online 6 June 2015

AMS 2000 subject classifications:
primary 60K25
secondary 60K20

Keywords:
$M/M/c$ queue
(s, S) production–inventory model
Lost sales
Regenerative process

ABSTRACT

We study an (s, S) production–inventory system with an attached Markovian service queue. A production facility gradually replenishes items in the inventory based on the (s, S) scheme, and the production process is assumed to be a Poisson process. In addition to the production–inventory system, c servers process customers that arrive in the system according to the Poisson process. The service times are assumed to be independent and identically distributed exponential random variables. The customers leave the system with exactly one item at the service completion epochs. If an item is unavailable, the customers cannot be served and must wait in the system. During this out-of-stock period, all newly arriving customers are lost. A regenerative process is used to analyze the proposed model. We show that the queue size and inventory level processes are independent in steady-state, and we derive an explicit stationary joint probability in product form. Probabilistic interpretations are presented for the inventory process. Finally, using mean performance measures, we develop cost models and show numerical examples.

1. Introduction

We study an (s, S) production–inventory system with an attached $M/M/c/\infty$ service queue. A production facility gradually replenishes items in the inventory based on the (s, S) scheme, and the production process is assumed to be a Poisson process. In the production–inventory system, c servers are assigned to serve customers that arrive in the system according to the Poisson process. The service times are assumed to be independent and identically distributed (i.i.d.) exponential random variables. The customers leave the system with exactly one item at the service completion epochs. If the inventory level drops to zero, the remaining customers must wait in the system to be served. During this out-of-stock period, all arriving customers are lost. Inventory models with an attached service queue originated from an assembly-like queue (or kitting queue) (Bozer & McGinnis, 1992; Bryznér & Johansson, 1995; Harrison, 1973; Lipper & Sengupta, 1986), where several types of parts are simultaneously combined, and a positive processing time is incurred only after all the types are gathered. Sigman and Simchi-Levi (1992) conducted the first extensive study on an inventory model with an attached service queue. Using a light traffic heuristic for an $M/G/1$ queue with limited inventory, they provided a closed-form expression for the average delay in terms of basic system parameters. Even though previous studies presented various mean performance measures for inventory models with an attached service queue, research on the joint probability of the queue length and

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http://dx.doi.org/10.1016/j.jkss.2015.05.002
1226-3192/

inventory level was rare, even in simple cases, because of the analytic difficulty due to strict dependency between the queue length and inventory processes.

Schwarz and Daduna (2006) and Schwarz, Sauer, Daduna, Kulik, and Szekli (2006) conducted extensive research on the joint probability of the $M/M/1$ service queue with an attached inventory controlled by various policies with exponential lead time and backordering. An inventory model with a service queue and lost sales was also studied by Saffari, Haji, and Hassanzadeh (2011). The previous models derived the stationary joint probabilities of the queue length and inventory level in product form, which implied that the inventory process is independent of the queue length process in steady-state. Recently, the (s, S) production–inventory model with an $M/M/1$ service queue was studied by Krishnamoorthy and Viswanath (2013) where the inventory items were gradually replenished by an internal production process instead of being instantly supplied with the lead time. In this paper, the inventory model with single server queue, proposed by Krishnamoorthy and Viswanath (2013), is extended to the multi-server case. The purposes of our research are to obtain an explicit stationary joint probability in product form using a regenerative method and to provide the probabilistic interpretations of the probability.

Krishnamoorthy and Viswanath (2013) applied a matrix theoretic approach to derive the stationary joint distribution to efficiently and computationally analyze the system. Using this approach, they obtained comprehensive performance measures and a cost model; however, they showed no probabilistic interpretation of the inventory process. Saffari, Asmussen, and Haji (2013) employed a regenerative process to analyze an inventory queue with a service queue to facilitate more stochastic interpretations. They considered an (r, Q) inventory model with an attached $M/M/1$ service queue, general lead times, and lost sales. Deviating from Krishnamoorthy and Viswanath (2013), they proved that the inventory level process was independent of the queue length process in steady-state and derived the stationary joint probability. It is important to note that the method used by Saffari et al. (2013) ultimately led to separate analyses of the inventory and queue length processes. Using a similar methodology, Baek and Moon (2014) introduced an extended model called the (r, Q) production–inventory system with an attached service queue. In this proposed model, items were assumed to be stocked by both an outside supplier and internal production.

In this paper, we use a regenerative process for the analysis. Our proposed method allows for more probabilistic interpretations of the proposed model. First, we prove the independence between the queue length process and the inventory level process of the proposed model. Later, the queue length process and inventory level process are separately analyzed. For the analysis of the queue length and inventory level processes, the traditional $M/M/c/\infty$ and $M/M/1/K$ queues are applied to derive the stationary probabilities. Furthermore, we discuss the applicability of the proposed method to other production–inventory models with a Markovian service queue and lost sales. Finally, we conclude that the steady-state inventory level process becomes identical to the process described by Krishnamoorthy and Viswanath (2013), and we determine the optimal conditions of the proposed model for each of the decision variables s and S.

The remainder of this paper is organized as follows. In Section 2, we introduce the proposed model in more detail and show the preliminary results. In Section 3, we analyze the proposed model, and in Section 4, we present a cost model and optimal conditions for selecting decision values s and S. We also compare the proposed cost function with the cost function of Krishnamoorthy and Viswanath (2013). Finally, conclusions are discussed in Section 5.

2. Preliminaries

In this section, we introduce the proposed model and review the busy period queue length formula for the $M/M/1/K$ system, which plays an important role in the analysis.

2.1. The model

We study an $M/M/c/\infty$ service queue with an attached production–inventory system and lost sales as shown in Fig. 1. There are c servers dedicated to serving customers one by one under the first-come, first-served (FCFS) discipline. The waiting room for customers is unlimited, and the size of the inventory is S. Customers arrive in the system according to a Poisson process with rate λ. During the period when there are items, arriving customers join the queue; however, all customers that arrive during an out-of-stock period are lost. The service times are assumed to be i.i.d. exponential random variables with mean $1/\mu$, and we assume $\lambda < c\mu$ for the ergodicity of the queue. A customer leaves the system with exactly one item from the inventory at his service completion epoch. When the inventory level reaches zero, the remaining customers in the system wait until the inventory is replenished.

Fig. 2 shows the sample path of the proposed model when $S = 6$ and $s = 4$. Let $N(t)$ and $J(t)$ be the number of customers and inventory level at time t, respectively.

The inventory items are gradually replenished by an internal production facility that is controlled by the (s, S) policy; specifically, the production facility is turned off as soon as the inventory level becomes S and reactivated if the level drops to s. In this way, the system alternates between a production period and a non-production period; the lengths of these periods are denoted by L_P and L_N, respectively. We assume that the internal production process follows a Poisson process with rate η. For analytic convenience, we assume that $\eta \neq \lambda$. In the lower part of Fig. 2, it is easy to see that L_N is the first passage time to level s from level S, and L_P is the first passage time to level S from level s.

Note that the queue length and the on-hand inventory processes are completely dependent; however, the inventory level process behaves like a regenerative process, where all of the non-production period starting points are the regeneration