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Exact likelihood inference of the exponential parameter under generalized Type II progressive hybrid censoring

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Type II progressive hybrid censoring schemes (PHCS) have become quite popular in reliability and lifetime-testing studies. However, the drawback of the Type II PHCS is that it might take a very long time in order to complete the life test. In this article, we propose generalized Type II PHCS which provide a guarantee of the time to complete the life test. Also, we consider both exact and approximate inferential procedures based on generalized Type II PHCS when the lifetime distribution of the test items follows an independent and identically distributed (iid) exponential distribution. Given by the exact conditional first moment, bias adjusted MLE is proposed and its distribution is discussed. The results of simulation studies and real data analysis were conducted to evaluate the performance of the proposed method.

1. Introduction

In reliability studies and life-testing experiments, the failure time data of experimental items are often not completely available. Reducing the cost and time associated with the experiments is crucial in statistical experiments with censored data. Among the different censoring schemes, the most popular censoring schemes are conventional Type I and Type II censoring schemes. However, if a researcher needs to withdraw live test items at times other than the termination time of the lifetime test, the conventional censoring schemes will not be of use. Intermediate withdrawal may be desirable when some of the live test items that are withdrawn early on can be used for some other tests, or when a compromise between the observation of at least some extreme life-times and reduced time of test is sought. Therefore, the censoring of test items at times other than the termination time may be desirable, as in the case of loss of connection with individuals under test or accidental breakage of test items. For these reasons and motivations, the progressive censoring is proposed (Balakrishnan & Aggarwala, 2000). The progressive censoring scheme may then be described as follows.

Consider n identical items are placed on a life test. Following the first observed failure, R_1 surviving items are withdrawn from the test at random. Following the second observed failure, R_2 surviving items are withdrawn from the test at random. This process continues until, following the mth observed failure, all the remaining $R_m = n - R_1 - \cdots - R_{m-1} - m$ items are withdrawn from the test. $X_{1:m:n} \leq X_{2:m:n} \leq \cdots \leq X_{m:m:n}$ denote the observed ordered failure times from the test. The joint probability density function (pdf) of $(X_{1:m:n}, X_{2:m:n}, \ldots, X_{i:m:n})$ $(i = 1, 2, \ldots, m)$ is given by

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(Balakrishnan & Aggarwala, 2000)

$$f(x_1, x_2, \ldots, x_i) = \left[\prod_{j=1}^{i}\sum_{k=j}^{m}(R_k+1)\right]\left[f(x_i)\{1-F(x_i)\}^{R_i^*-1}\right]\prod_{j=1}^{i-1}f(x_j)[1-F(x_j)]^{R_j}, \tag{1}$$

where $R_i^* = \sum_{k=i}^{m}(R_k+1), \ -\infty < x_1 < x_2 < \cdots < x_i < \infty$.

Kundu and Joarder (2006) also considered a PHCS in which the lifetime test is terminated at a $\min\{X_{m:m:n}, T\}$, where $T \in (0, \infty)$ and integer m are pre-assigned. Under this PHCS, the time on test will be no more than T. Childs, Chandrasekar, and Balakrishnan (2007) refer to this scheme as Type I PHCS. They derived the exact conditional distribution of the maximum likelihood estimator (MLE) of the exponential parameter as well as a confidence interval (CI) for exponential parameter under Type I PHCS. Some recent studies on Type I PHCS have been carried out by many authors including Lin, Chou, and Huang (2012), Lin, Huang, and Balakrishnan (2013) and Wu, Shi, and Sun (2014).

However, the drawback of the Type I PHCS is that there is a possibility that very few failures may occur before time T. For this motivation, Cho, Sun, and Lee (2015b) proposed a new generalized PHCS, referred to as generalized Type I PHCS, which guarantees a fixed number of failures. In generalized Type I PHCS, the termination time is $\max\{X_{k:m:n}, \min\{X_{m:m:n}, T\}\}$, where $T \in (0, \infty)$ and integer k and $m \in \{1, 2, \ldots, n\}$ are pre-assigned. They also derived the exact distribution of the MLE of the exponential parameter and a lower confidence bound for exponential parameter under generalized Type I PHCS. Cho, Sun, and Lee (2015a) also derived the Bayes estimation of the entropy function of the Weibull distribution under generalized Type I PHCS.

Also, Childs et al. (2007) considered a Type II PHCS in which the lifetime test is terminated at a $\max\{X_{m:m:n}, T\}$, where $T \in (0, \infty)$ and integer m are pre-assigned. If $X_{m:m:n} < T$, then instead of terminating the test by withdrawing the remaining R_m items after the mth failure, researcher continue to observe failures up to time T. Moreover, for the purpose of increasing the efficiency of statistical analysis as well as saving the total test time, Ng, Kundu, and Chan (2009) introduced an adaptive Type II PHCS. If $X_{m:m:n} < T$, the experiment stops at the time $X_{m:m:n}$. Otherwise, once the experimental time passed time T but the number of observed failures has not reached m, researcher does not withdraw any items at all except for the time of the mth failure where all remaining surviving items are removed. Some recent studies on Type II PHCS and adaptive Type II PHCS have been carried out by many authors including Chan, Ng, and Su (2015), Hemmati and Khorram (2013) and Lin and Huang (2012).

Though the Type II PHCS and adaptive Type II PHCS guarantee a specified number of failures, it has the drawback that it might take a very long time to observe mth failures and complete the life test. In this reason, we propose a generalized Type II PHCS in which the experiment is guaranteed to terminate at a pre-fixed time. These are designed to fix the drawbacks inherent in the Type II PHCS. The life-testing experiment based on the proposed censoring scheme can save both the total time on tests and the cost. Such a censoring scheme may arise in a situation when the experimenter has prepaid for the use of the testing facility for T units of time. However, in Type II PHCS, there is a disadvantage that far fewer than m failures may be observed. The detailed description will be described in the next section. In Table 1, we present a comparison of Type II PHCS and generalized Type II PHCS for some selected choices of schemes. The values of the expected length and expected number of failures for the two PHCS in the table give supportive evidence to the advantage and disadvantage of the generalized Type II PHCS. In Table 1, for simplicity in notation, we denote the scheme $(0, 0, \ldots, n-m)$ as $((m-1)*0, n-m)$, to give you an idea, $(10*0)$ and $(3*0, 2, 2, 0)$ denote the progressively censoring schemes $(0, 0, \ldots, 0)$ and $(0, 0, 0, 2, 2, 0)$, respectively. In this article, we assume that the lifetimes of observations are independent and identically distributed (i.i.d.) with an exponential distribution. Under exponential data assumption, we consider the exact and approximate conditional statistical inference based on generalized Type II PHCS.

The rest of the paper is organized as follows. In Section 2, we consider the case of an exponential distribution under generalized Type II PHCS and discuss the MLE of the exponential parameter. Moreover, we derive the exact conditional moment generating function (mgf) of the MLE. We then derive the exact conditional distribution of the MLE. In Section 3, based on the exact conditional mean, bias adjusted estimate is proposed. In Section 4, we use exact conditional distribution to obtain exact conditional CI for the exponential parameter. CIs of the exponential parameter based on asymptotic distribution of the MLE and the bias adjusted estimator are also developed. A Monte Carlo simulation of inferential procedures is carried out in Section 5. Also, Real data analysis has been analyzed in Section 5. The method of finding the optimal censoring scheme is discussed in Section 6 and finally we conclude the paper in Section 7.

2. Model, likelihood and conditional MLE

Consider a lifetest in which n identical items are put on test. Then, the generalized Type II PHCS may be described as follows. The integer m, times T_1 and T_2 are pre-assigned such that $m \le n$ and $0 < T_1 < T_2 < \infty$, and also $R_1, R_2, \ldots, R_m$ are pre-assigned integers satisfying $\sum_{i=1}^{m} R_i + m = n$. Let D_1 and D_2 denote the number of observed failures up to time T_1 and T_2, respectively. Also, let d_1 and d_2 are the observed value of D_1 and D_2, respectively. At the time of first observed failure, R_1 of the remaining items are withdrawn from the test at random. Following the second observed failure, R_2 of the remaining items are withdrawn and so on. If $X_{m:m:n} < T_1$, then instead of terminating the test by withdrawing the remaining R_m items after the mth failure, we continue to observe failures (without any further withdrawals) up to time T_1. Therefore,