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Analyzing an infinite buffer batch arrival and batch service queue under batch-size-dependent service policy

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ABSTRACT

In this paper, we investigate an infinite-buffer queue with batch-arrival and batch-service wherein a single server operates under random serving capacity rule with service time dependent on the size of the batch under the service. First, we derive the probability generating function of state probabilities at service completion epoch, from which an entire spectrum regarding queue-length at various epochs is extracted. Using the departure epoch probabilities, we establish a stable relationship between departure and random epochs probabilities based on 'rate in = rate out' approach. Further, random epoch probabilities are used to obtain pre-arrival epoch probabilities. Finally, we illustrate our analytical results by means of numerical computation which includes the case of multiple roots.

1. Introduction

In conventional queueing models customers arrive singly and are served individually. However, in numerous real-life circumstances it is often observed that customers arrive in batches and are also served in batches. Such practical scenario can be adequately framed by batch-arrival and batch-service queues (traditionally called bulk-queues).

Now a days, on account of the wide range of applications, bulk-queues with batch-size-dependent service have attracted significant attention to the researchers. A textbook example of such queue consists of system where the server is a human being and the amount of work may directly influence the serving capability of the server. Another example includes modern wireless communication system, dealing with multimedia type of data, where requests arrive in batches of random sizes and service are rendered in varying size of batches. There exist manufacturing systems where jobs are usually sorted out in batches, and these batches are forwarded to different station where they are re-grouped for another batch processing. Such intermediate station may be considered as batch-arrival and batch-service queueing model. More interesting examples include group-testing policy for detecting HIV in the blood samples, see Abolnikov and Dukhovny (2003).

Bar-Lev, Parlar, Perry, Stadje, and Van der Duyn Schouten (2007) considered an $M/G^{(a,b)}/1$ queue with batch-size-dependent service. After deriving probability generating function (p.g.f.) of queue length distribution at departure epoch of a batch, they stayed away from inverting it due to the complexity involved in the inversion. However, they truncated the

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transition probability matrix (TPM) into a finite state space and obtained queue-length distribution by solving finite number of system of equations. The significant feature in their study is the detailed presentation of average profit and cost functionals from a purely economic point of view. They have also shown that all the functionals can be expressed in terms of the steady-state distribution of the embedded Markov chain. Later, by considering the p.g.f. given by Bar-Lev et al. (2007, p. 230) and Chaudhry and Gai (2012) successfully inverted it using the method of roots and presented closed-form formulae in terms of roots of the queue length distribution at departure epoch. They also discussed both the $M/D_j^{(a,b)}/1$ and $M/D_j^{(a,b)}/1/B+b$ queues. However, none of them obtained queue-length distribution at random epoch.

As a counterpart of continuous-time queue, in discrete-time queue there exists only a few literature on batch-size-dependent batch-service queue. A sequence of papers, considered by Claeys, Steyaert, Walraevens, Laevens, and Bruneel (2013a,b); Claeys, Walraevens, Laevens, and Bruneel (2010) consists of batch-service queue with a general dependency between service time and batch size under service. In Claeys et al. (2010), they considered $Geo^X/G^{(l,c)}/1$ queue and obtained joint p.g.f. of the queue content and the server content (the number of customers with the server). Then from the joint p.g.f. they extracted the p.g.f.s of queue content and server content. Further, under discrete-batch Markovian arrival process (D-BMAP) (Claeys et al., 2013a), they derived joint vector generating function of the queue content, the server content, and the remaining service time of the batch under service and then extracted marginal p.g.f.s for several admissible quantities such as queue content when server is inactive, server content at the end of service, etc. They also elaborated upon the influence of correlation of the arrival process on the mean system content. Furthermore, in Claeys et al. (2013b) they approximated tail probabilities of the customer delay by considering $Geo^X/G^{(l,c)}/1$ queue. They also illustrated that neglecting of batch-size-dependent service times can lead to a devastating inaccuracy of the approximation of the tail probabilities.

It is widely recognized that many practical life situations deal with the finite-buffer queue. Getting motivated from this fact, few authors have analyzed batch-size-dependent service queue when buffer size is finite. Banerjee and Gupta (2012) considered $M/G_r^{(a,b)}/1/N$ queue where the service time depends on the service batch size and obtained joint distribution of queue content and server content. Further, Banerjee, Gupta, and Sikdar (2013) analyzed $M^X/G_r^Y/1/N$ queue under random service capacity. Recently, Germs and van Foreest (2013) analyzed a more complex group-arrival and batch-service finite-buffer queueing model: $M(n)^{X(n)}/G(n)^{Y(n)}/1/K+B$, where (i) arrival rate depends on queue length (ii) both arriving group size and service batch size depends on queue length (iii) service time also depends on service batch size. They analyzed this model using semi-regenerative process and developed a numerically stable method to calculate distribution of the queue length and relevant performance measures.

Taking into consideration of importance of bulk-queues with batch-size-dependent service, in this paper we consider an infinite-buffer $M^X/G_n^Y/1$ queue where arriving batch sizes as well as serving capacity of the server are random and service time depends on the batch size under service. To deal with the real life applications, random capacity of the server is assumed to be of finite support. This paper mainly focuses on analyzing this queue analytically and computationally. For this purpose, from the associated Markov chain we first obtain the TPM from which the p.g.f. of the queue length at departure epoch is derived. We then present a closed-form expression of the departure epoch probabilities in terms of roots. This also includes the investigations about the non-zero multiple roots occurring in the characteristic equation. Further, in order to obtain random and pre-arrival epoch probabilities, two stable relationships are established between departure and random epoch probabilities, and between random and pre-arrival epoch probabilities. Finally, illustration through some numerical examples are made out for a variety of service time distribution, viz. exponential, Erlang, hyper-exponential, deterministic.

The outline of the rest of this paper is as follows: after giving the formal description of the model in Section 2, we use the embedded Markov chain technique to derive the queue-length distribution at service completion epoch in Section 3. Section 4 is assigned to obtain the random epoch probabilities by establishing a relation between the queue length distributions at departure and random epochs. A relationship between the queue length distributions at random and pre-arrival epochs is given in Section 5, while Section 6 includes some special cases of our model. A computational experience for several service time distributions are presented in Section 7. Some conclusions are drawn in Section 8 followed by the references.

2. Model description

This section specifies the details of the queueing model.

- Groups of customers arrive according to Poisson process with rate λ. The arriving group sizes are independently identically distributed random variable with probability distribution $P(X = m) = g_m,\ m \in \mathbb{N}$, where X is generic group size with finite mean $E(X) = \bar{g}$ and associated p.g.f. $G(z) = \sum_{m=1}^{\infty} g_m z^m$.
- At each service commencement, the server has random serving capacity Y, governed by the probability distribution $P(Y = i) = y_i,\ i = 1, 2, \ldots, B$, with $y_B > 0$, with its associated p.g.f. $Y(z) = \sum_{i=1}^{B} y_i z^i$ and mean batch size $\bar{y}$, where B is the maximum serving capacity of the server. At the beginning of a service, if the queue length is less than the service capacity i, then the server makes the queue empty and begin service without utilizing the surplus capacity, see Chang and Choi (2006) and Germs and Van Foreest (2010). If the server finds no customer in the queue at the service completion epoch of a batch, it enters into the idle phase. An arriving customer cannot join on the ongoing service even if the server has a free capacity.