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Neutral stochastic partial differential equations with delay driven by Rosenblatt process in a Hilbert space ☆

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ABSTRACT

In this paper, we prove an existence and uniqueness result of the mild solution for a neutral stochastic partial differential equations with finite delay driven by Rosenblatt process in a real separable Hilbert space. An example is provided to illustrate the effectiveness of the proposed result.

1. Introduction

Because of its wide application in various sciences such as physics, mechanical engineering, control theory and economics, the theory of stochastic partial differential equations (SPDEs) has been investigated by many authors and has already achieved fruitful results (see for example Chow, 1982, Hu & Ren, 2010, Luo & Liu, 2008 and the references therein). Particularly, the investigation for SPDEs with delay driven by fractional Brownian motion has attracted considerable attentions of researchers and many qualitative theories for the solutions of this kind have been derived. For example, Ferrante and Rovira (2006) studied the existence and regularity of the solutions by using the Skorohod integral based on the Malliavin calculus. Neuenkirch, Nourdin, and Tindel (2008) studied the problem by using rough path analysis. Ferrante and Rovira (2010) studied the existence and convergence when the delay goes to zero by using the Riemann–Stieltjes integral. Using also the Riemann–Stieltjes integral, Boufoussi and Hajji (2011) and Boufoussi, Hajji, and Lakhel (2012) proved the existence and uniqueness of a mild solution and studied the dependence of the solution on the initial condition in finite and infinite dimensional space. Boufoussi and Hajji (2012) studied the existence, uniqueness and asymptotic behavior of mild solutions for the neutral stochastic differential equation with finite delay. Recently, Caraballo, Garrido-Atienza, and Taniguchi (2011) discussed the existence, uniqueness and exponential asymptotic behavior of mild solutions.

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On the other hand, to the best of our knowledge, there is no work done on neutral stochastic partial differential equations with delay driven by a Rosenblatt process. Motivated by the aforementioned works, as a first attempt, we will consider the following neutral stochastic partial differential equations with delay driven by a Rosenblatt process of the form:

$$\begin{cases} \mathrm{d}[x(t) + G(t, x(t-u(t)))] = [Ax(t) + f(t, x(t-r(t)))]\,\mathrm{d}t + \sigma(t)\,\mathrm{d}Z_H(t), & t \in J := [0, T], \\ x(t) = \phi(t), \quad -\tau \le t \le 0, \end{cases} \tag{1.1}$$

in a real separable Hilbert space U with inner product $\langle\cdot,\cdot\rangle$ and norm $\|\cdot\|$, where A is the infinitesimal generator of an analytic semigroup of bounded linear operators, $(S(t))_{t\ge0}$, in a Hilbert space U. $Z_H(t)$ is a Rosenblatt process with parameter $H \in (\frac{1}{2}, 1)$ on a real and separable Hilbert space $(K, \|\cdot\|_K, \langle\cdot,\cdot\rangle_K)$. $u(t), r(t) : [0, +\infty) \to [0, \tau]$ $(\tau > 0)$ are continuous, $G, f : [0, +\infty) \times U \to U$ and $\sigma : [0, +\infty) \to L_Q(K, U)$ are given functions to be specified later, here, $L_Q(K, U)$ denotes the space of all Q-Hilbert–Schmidt operators from K into U, which will be defined in the next section. The initial data $\phi = \{\phi(t) : -\tau \le t \le 0\} \in \mathcal{C}([-\tau, 0], L^2(\Omega, U))$. Our work is devoted to prove the existence and uniqueness of the mild solution for the system (1.1). The result is obtained by means of the Banach fixed point theorem. An example is given to illustrate the theory.

The paper is organized as follows. In Section 2, we introduce some preliminaries. In Section 3, we prove the existence and uniqueness of a mild solution for system (1.1). An example is provided in the last section to illustrate the theory.

2. Preliminaries

In this section, we provide some preliminaries needed to establish our main results. Throughout this paper, unless otherwise specified, we use the following notations. Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a complete probability space and for $t \ge 0$, $\mathcal{F}_t$ denote the σ-field generated by $\{Z_H(s), s \in [0, t]\}$ and the $\mathbb{P}$-null sets. The notation $L^2(\Omega, U)$ stands for the space of all U-valued random variables x such that $E\|x\|^2 = \int_\Omega \|x\|^2\,\mathrm{d}\mathbb{P} < \infty$. For $x \in L^2(\Omega, U)$, let $\|x\|_2 = \left(\int_\Omega \|x\|^2\,\mathrm{d}\mathbb{P}\right)^{1/2}$. It is easy to check that $L^2(\Omega, U)$ is a Hilbert space equipped with the norm $\|\cdot\|_2$. Let $L(K, U)$ denotes the space of all bounded linear operators from K to U, we abbreviate this notation to $L(K)$ whenever $U = K$ and $Q \in L(K)$ represents a non-negative self-adjoint operator.

Let K be a separable Hilbert space and $L_2^0 = L^2(K, U)$ be a separable Hilbert space with respect to the Hilbert–Schmidt norm $\|\cdot\|_{L_2^0}$. Let $L_Q^0(K, U)$ be the space of all $\psi \in L(K, U)$ such that $\psi Q^{\frac{1}{2}}$ is a Hilbert–Schmidt operator. The norm is given by $\|\psi\|^2_{L_Q^0} = \left\|\psi Q^{\frac{1}{2}}\right\|^2 = \mathrm{tr}(\psi Q \psi^*)$. Then ψ is called a Q-Hilbert–Schmidt operator from K to U. In the sequel, $L_2^0(\Omega, U)$ denotes the space of $\mathcal{F}_0$-measurable, U-valued and square integrable stochastic processes.

2.1. Rosenblatt process

In this subsection, we recall some basic knowledge on the Rosenblatt process as well as the Wiener integral with respect to it.

Consider $(\xi_n)_{n\in\mathbb{Z}}$ a stationary Gaussian sequence with mean zero and variance 1 such that its correlation function satisfies that $R(n) := E(\xi_0\xi_n) = n^{\frac{2H-2}{k}}L(n)$, with $H \in (\frac{1}{2}, 1)$ and L is a slowly varying function at infinity. Let g be a function of Hermite rank k, that is, if g admits the following expansion in Hermite polynomials

$$g(x) = \sum_{j\ge0} c_j H_j(x), \quad c_j = \frac{1}{j!}E(g(\xi_0 H_j(\xi_0))),$$

then $k = \min\{j | c_j \ne 0\} \ge 1$, where $H_j(x)$ is the Hermite polynomial of degree j given by $H_j(x) = (-1)^j e^{\frac{x^2}{2}} \frac{d^j}{dx^j} e^{-\frac{x^2}{2}}$. Then, the Non-Central Limit Theorem (see, for example, Dobrushin & Major, 1979) says $\frac{1}{n^H}\sum_{j=1}^{[nt]} g(\xi_j)$ converges as $n \to \infty$, in the sense of finite dimensional distributions, to the process

$$Z_H^k(t) = c(H, k)\int_{\mathbb{R}^k}\int_0^t \left(\prod_{j=1}^k (s - y_j)_+^{-\left(\frac{1}{2}+\frac{1-H}{k}\right)}\right)\mathrm{d}s\mathrm{d}B(y_1)\ldots\mathrm{d}B(y_k), \tag{2.1}$$

where the above integral is a Wiener–Itô multiple integral of order k with respect to the standard Brownian motion $(B(y))_{y\in\mathbb{R}}$ and $c(H, k)$ is a positive normalization constant depending only on H and k. The process $(Z_H^k(t))_{t\ge0}$ is called as *the Hermite process* and it is H self-similar in the sense that for any $c > 0$, $(Z_H^k(ct)) \overset{d}{=} (c^H Z_H^k(t))$ and it has stationary increments.

The most studied Hermite process is of course the fractional Brownian motion (which is obtained in (2.1) for $k = 1$) due to its large range of applications. When $k = 2$, the process given by (2.1) is known as the Rosenblatt process (it has actually been named in this way by Taqqu in Taqqu (1975)). This process is of interest in practical applications because of its self-similarity, stationarity of increments and long range dependence (see Tindel, Tudor, & Viens, 2003). However, it is not Gaussian. Actually the very large utilization of the fractional Brownian motion in practice (hydrology, telecommunications) are due to