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Bivariate zero truncated Poisson INAR(1) process

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ABSTRACT

In this paper, we propose a new stationary bivariate first order integer-valued autoregressive (BINAR(1)) process with zero truncated Poisson marginal distribution. Some properties about this process are considered, such as probability generating function, autocorrelations, expectations and covariance matrix under conditional and unconditional situation. We also establish the strict stationarity and ergodicity of the process. Estimators of unknown parameters are derived by using Yule–Walker, conditional least squares and maximum likelihood methods. The performance of the proposed estimation procedures are evaluated through Monte Carlo simulations. An application to a real data example is also provided.

1. Introduction

In recent years, there has been a growing interest in integer-valued time series, which often occur in many fields, such as communication, insurance theory, reliability theory, and meteorology etc. Many results on univariate integer-valued time series data have been published. For example, Al-Osh and Alzaid (1987) proposed the first-order non-negative integer valued autoregressive (INAR(1)) process; Bakouch and Ristić (2010) studied a new INAR(1) model with zero truncated Poisson marginal; Zhang, Wang, and Zhu (2010) introduced a pth-order integer valued autoregressive processes with signed generalized power series thinning operator; Zhang and Wang (2015), Zhang, Wang, and Zhu (2012) and Zheng, Basawa, and Datta (2006, 2007) presented some results on random coefficient integer-valued autoregressive processes; Li, Wang, and Zhang (2015) proposed a first-order mixed integer-valued autoregressive process with zero-inflated generalized power series innovations. Extensions of the above INAR-type processes to the bivariate case were introduced in the literature. For example, Pedeli and Karlis (2011) extended the INAR(1) model to bivariate situation and introduced some applications; Ristić and Nastić (2012) discussed the bivariate INAR(1) model with geometry marginal; Pedeli and Karlis (2013) studied the estimation of bivariate INAR(1) with bivariate Poisson marginals; Scotto, Weiß, Silva, and Pereira (2014) proposed some bivariate binomial autoregressive models. For more details on integer-valued time series, we refer to Fokianos (2011), Turkman, Scotto, and Bermudez (2014) and Weiß (2008).

As we know, zero-truncated distributions play an important role in practice. Plackett (1953) introduced the Poisson distribution with zero truncation and discussed some properties and estimations; Cohen (1960) presented an extension of

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this zero truncated Poisson distribution; Zelterman (1988) studied the robust estimation in truncated discrete distributions with application to capture–recapture experiments; Heijden, Cruyff, and Houwelingen (2003) used the truncated Poisson regression model to estimate the size of a criminal population from police records; Ghitany, Al-Mutairi, and Nadarajah (2008) investigated zero-truncated Poisson–Lindley distribution and its application; For count time series data, Bakouch and Ristić (2010) introduced a stationary integer-valued autoregressive process of the first order with zero truncated Poisson marginal distribution. To the best of our knowledge, there is no existing result on bivariate zero truncated INAR process, thus it is desired to extend Bakouch and Ristić (2010)'s results to the bivariate model.

The remainder of this paper is organized as follows. In Section 2, we construct the new bivariate zero truncated INAR process with Poisson marginal distribution. In Section 3, some statistical properties of the process have been discussed. Also the strict stationarity and ergodicity are also obtained. In Section 4, we estimate the parameters of interest by the methods of Yule–Walker, conditional least squares and maximum likelihood methods, respectively. Section 5 presents some simulation results of the proposed procedures. In Section 6, we apply our model to a real data example. Some conclusion remarks are given in Section 7. All proofs are relegated to Appendix.

2. Construction of the process

According to Johnson, Kotz, and Balakrishnan (1997), we give the definition of bivariate zero truncated Poisson distribution which will be used to construct the new process.

Definition 2.1. A non-negative integer-valued random vector (X, Y) is said to follow a bivariate zero truncated Poisson distribution if the probability mass function is given by

$$P(X=i, Y=j) = \frac{e^{-\lambda_1-\lambda_2+\phi}}{1-e^{-\lambda_1-\lambda_2+\phi}} \times \frac{(\lambda_1-\phi)^i}{i!} \times \frac{(\lambda_2-\phi)^j}{j!} \times \sum_{k=0}^{s} \binom{i}{k}\binom{j}{k} k! \left[\frac{\phi}{(\lambda_1-\phi)(\lambda_2-\phi)}\right]^i,$$

where $(i,j) \in \mathbb{N}_0^2 \setminus \{(0,0)\}, s = min(i,j), \lambda_1 > 0, \lambda_2 > 0$ and $\phi \in [0, \min(\lambda_1,\lambda_2)]$. Denote this distribution as $BZTP(\lambda_1,\lambda_2,\phi)$.

Now we consider a stationary bivariate integer-valued autoregressive process $\mathbf{X}_t = (X_{1t}, X_{2t})^T$ with $BZTP(\lambda_1,\lambda_2,\phi)$ marginal distribution, which is defined as

$$\mathbf{X}_t = \begin{cases} \mathbf{A} \circ \mathbf{X}_{t-1} + \boldsymbol{\varepsilon}_t, & \text{with probability } 1 - e^{-(\lambda_1+\lambda_2-\phi)}, \\ \boldsymbol{\varepsilon}_t, & \text{with probability } e^{-(\lambda_1+\lambda_2-\phi)}, \end{cases} \quad t \in \mathbb{Z}, \tag{2.1}$$

where "$\circ$" is the thinning operator defined in Steutel and Van Harn (1979), $\mathbf{A} = diag\{\alpha_1, \alpha_2\}$ with $\alpha_i \in (0,1),\ i = 1, 2$; $\{\boldsymbol{\varepsilon}_t\}$ is a sequence of i.i.d. bivariate random vectors.

First, we need to derive the distribution of $\boldsymbol{\varepsilon}_t$. Let $\varphi_{\mathbf{X}_t}(s_1,s_2)$ and $\varphi_{\boldsymbol{\varepsilon}_t}(s_1,s_2)$ denote the joint probability generating functions of $\mathbf{X}_t$ and $\boldsymbol{\varepsilon}_t$, respectively. From the definition of joint probability generating function, we can obtain that

$$\varphi_{\mathbf{X}_t}(s_1,s_2) = E{s_1}^{X_{1t}}{s_2}^{X_{2t}} = \frac{e^{\lambda_1 s_1+\lambda_2 s_2+\phi(s_1s_2-s_1-s_2)}-1}{e^{\lambda_1+\lambda_2-\phi}-1}. \tag{2.2}$$

Then by the stationarity of BINAR(1) process, it is easy to derive that

$$\begin{aligned}\varphi_{\boldsymbol{\varepsilon}_t}(s_1,s_2) &= \frac{\varphi_{X_t}(s_1,s_2)}{e^{-(\lambda_1+\lambda_2-\phi)} + (1-e^{-(\lambda_1+\lambda_2-\phi)})\varphi_{X_t}((1-\alpha_1+s_1\alpha_1),(1-\alpha_2+s_2\alpha_2))} \\ &= \frac{e^{\lambda_1 s_1+\lambda_2 s_2+\phi(s_1s_2-s_1-s_2)}-1}{(1-e^{-(\lambda_1+\lambda_2-\phi)})e^{\lambda_1(1-\alpha_1+s_1\alpha_1)+\lambda_2(1-\alpha_2+s_2\alpha_2)+\phi[(1-\alpha_1+s_1\alpha_1)(1-\alpha_2+s_2\alpha_2)-(1-\alpha_1+s_1\alpha_1)-(1-\alpha_2+s_2\alpha_2)]}}.\end{aligned} \tag{2.3}$$

Because $\varphi_{\boldsymbol{\varepsilon}_t}(1,1) = \sum_i\sum_j P(\varepsilon_{1t}=i, \varepsilon_{2t}=j) = 1$, it follows that (2.3) is a proper probability generating function of $\boldsymbol{\varepsilon}_t$. Moreover, after some tedious calculations, we can obtain the joint probability mass function of $\boldsymbol{\varepsilon}_t$ as follows,

$$\begin{aligned}P(\varepsilon_{1t}=i, \varepsilon_{2t}=j) = \frac{e^{\lambda_1\alpha_1+\lambda_2\alpha_2-\phi\alpha_1\alpha_2}}{e^{\lambda_1+\lambda_2-\phi}-1} &\times \left\{\frac{[\lambda_1(1-\alpha_1)-\phi(1-\alpha_1\alpha_2)]^i}{i!}\frac{[\lambda_2(1-\alpha_2)-\phi(1-\alpha_1\alpha_2)]^j}{j!}\right. \\ &\times \sum_{k=0}^{\min(\varepsilon_{1t},\varepsilon_{2t})}\binom{i}{k}\binom{j}{k}k!\frac{[\phi(1-\alpha_1\alpha_2)]^k}{[\lambda_1(1-\alpha_1)-\phi(1-\alpha_1\alpha_2)]^k[\lambda_2(1-\alpha_2)-\phi(1-\alpha_1\alpha_2)]^k} \\ &- \frac{(\phi\alpha_1\alpha_2-\lambda_1\alpha_1)^i}{i!}\frac{(\phi\alpha_1\alpha_2-\lambda_2\alpha_2)^j}{j!} \\ &\left.\times \sum_{k=0}^{\min(\varepsilon_{1t},\varepsilon_{2t})}\binom{i}{k}\binom{j}{k}k!\frac{(-\phi\alpha_1\alpha_2)^k}{(\phi\alpha_1\alpha_2-\lambda_1\alpha_1)^k(\phi\alpha_1\alpha_2-\lambda_2\alpha_2)^k}\right\}.\end{aligned} \tag{2.4}$$