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A test for the increasing convex order based on the cumulative residual entropy

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ABSTRACT

The complete cumulative residual entropy can be generalized to the incomplete cumulative residual entropy (ICRE). In this paper, we introduce a partial ordering in terms of ICRE. The relationship between this ordering and some important orderings of lifetime distributions are investigated. We use this order to establish a test statistic for testing the stochastic equality against increasing convex order alternative. The performance of the test statistic is evaluated using a simulation study. Finally, a numerical example illustrating the theory is also given.

1. Introduction

Let X be a non-negative random variable with distribution function F and survival function $\bar{F} = 1 - F$. The uncertainty contained in a random variable, since its first mathematical formulation by Shannon (1948), has been defined in several formula. Recently, Rao, Chen, Vemuri, and Wang (2004) define a new uncertainty measure, the (complete) Cumulative Residual Entropy (CRE), through

$$\mathcal{E}(X) = -\int_0^\infty \bar{F}(x) \log \bar{F}(x) dx = \int_0^\infty \bar{F}(x) \Lambda(x) dx, \tag{1}$$

where $\Lambda(x) = -\log \bar{F}(x)$ is the cumulative hazard function. Properties of the CRE can be found in Rao (2005), Di Crescenzo and Longobardi (2009), and Navarro, del Aguila, and Asadi (2010). Asadi and Zohrevand (2007) considered the corresponding dynamic properties of the CRE corresponding to the residual lifetime variable. Applications of CRE to image alignment and measurements of similarity between images can also be found in Wang and Vemuri (2007) and references therein.

The "complete" CRE function $\mathcal{E}(X)$ can be generalized to the incomplete CRE function $\mathcal{E}(X, t)$ such that $\mathcal{E}(X, 0) = \mathcal{E}(X)$. This "upper" incomplete CRE (ICRE) function is defined as follows:

$$\mathcal{E}(X, t) = \int_t^\infty \bar{F}(x) \Lambda(x) dx. \tag{2}$$

In the literature several concepts of stochastic ordering among random variables have been considered. These concepts are useful in modeling for reliability and economics applications. For example, stochastic orders have shown to be useful

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notions in the comparison and analysis of poverty and income inequality, risk analysis or portfolio insurance. For more examples of applications and a comprehensive discussion on the stochastic ordering, we refer the reader to Berrendero and Carcamo (2011), Shaked and Shanthikumar (2007). In this paper, we use (2) to introduce a new stochastic ordering of random variables which can be used in the stochastic comparison of two random variables X and Y (or equivalently their respective survival function $\bar{F}$ and $\bar{G}$). The relation between this order and other famous stochastic orders is studied. We use the relationship between the ICRE order and the increasing convex order to propose a statistical test for testing the stochastic equality against increasing convex order alternative.

The rest of the paper is organized as follows. In Section 2 we give a new stochastic ordering of random variables based on ICRE and its relationship with the usual stochastic order and the increasing convex order. In Section 3 we apply ICRE order to construct a test for the increasing convex order. Section 4 is devoted to the simulation results and a numerical example and finally, some concluding remarks are given in Section 5.

2. Comparison based on ICRE

Let X and Y be two random variables with survival functions $\bar{F}$ and $\bar{G}$, cumulative hazard functions Λ_X and Λ_Y, and finite CREs $\mathcal{E}(X)$ and $\mathcal{E}(Y)$, respectively. Then, we have the following definition.

Definition 2.1. X is said to be less than Y in incomplete cumulative residual entropy (denoted by $X \leq_{ine} Y$) if, for all $t \geq 0$,

$$\mathcal{E}(X,t) \leq \mathcal{E}(Y,t). \tag{3}$$

Observe that $\mathcal{E}(X,t)$ can also be written as

$$\mathcal{E}(X,t) = E[\psi(t,X)I(X>t)], \tag{4}$$

where

$$\psi(t,x) = \int_t^x \Lambda_X(y)dy.$$

It is interesting to note that if X and Y are two exponential random variables with hazard rates λ_1 and λ_2, respectively, and if $\lambda_1 \geq \lambda_2$ then $X \leq_{ine} Y$. Now, we give the definition of other partial orderings which are well known in the literature (see, e.g. Shaked & Shanthikumar, 2007) and we shall use in this paper.

Definition 2.2. (a) The random variable X is said to be smaller than Y in the usual stochastic order (denoted by $X \leq_{st} Y$) if $\bar{F}(t) \leq \bar{G}(t)$ for all t.

(b) The random variable X is said to be smaller than Y in the increasing convex order (denoted by $X \leq_{icx} Y$) if $\int_t^\infty \bar{F}(x)dx \leq \int_t^\infty \bar{G}(x)dx$ for all t.

The following results give the relationship between our ordering and the orderings described in Definition 2.2. First, the theorem below shows that the usual stochastic order implies the ICRE order.

Theorem 2.3. *If $X \leq_{st} Y$, then $X \leq_{ine} Y$.*

Proof. It is clear that, for any given t, the function $h(x) = \psi(x,t)I(x>t)$ in (4) is increasing in x. The result now follows from the fact that $X \leq_{st} Y$ if, and only if, $E[h(X)] \leq E[h(Y)]$ for all increasing functions h for which the expectations exist.

The following example shows that the converse of Theorem 2.3 is not valid.

Example 2.4. Let X be distributed as Weibull with survival function

$$\bar{F}(x) = e^{-(\frac{x}{\beta})^\alpha}, \quad x>0,\ \alpha,\beta>0,$$

and Y be distributed as Pareto with survival function

$$\bar{G}(x) = \left(\frac{b}{x+b}\right)^a, \quad x>0,\ a>1, b>0.$$

Then,

$$\mathcal{E}(X,t) = \frac{\beta}{\alpha}\Gamma\left(\frac{\alpha+1}{\alpha},\left(\frac{t}{\beta}\right)^\alpha\right),$$

and

$$\mathcal{E}(Y,t) = \frac{ab}{(a-1)^2}\Gamma\left(2,(a-1)\log\frac{t+b}{b}\right),$$