Journal of the Korean Statistical Society 44 (2015) 516–529

Contents lists available at ScienceDirect

Journal of the Korean Statistical Society

journal homepage: www.elsevier.com/locate/jkss

Stationary distribution of the surplus in a risk model with dividends and reinvestments

Sunggon Kim [a], Eui Yong Lee [b,*]

[a] Department of Statistics, University of Seoul, Seoul, 130-743, Republic of Korea

[b] Department of Statistics, Sookmyung Women's University, Seoul, 140-742, Republic of Korea

ARTICLE INFO

Article history:
Received 3 April 2014
Accepted 15 January 2015
Available online 7 February 2015

AMS 2000 subject classifications:
primary 60J25
secondary 60G10

Keywords:
Risk model
Stationary distribution
Surplus process
Level crossing argument
Impulse control

ABSTRACT

A continuous time risk model with dividends and reinvestments is considered. We obtain an explicit formula of the stationary distribution of the surplus and the expected time to ruin after a reinvestment by adopting the level crossing argument. We also propose a scheme to approximate the stationary distribution of the surplus. As an example, we consider the case when the claims are exponentially distributed, Erlang distributed, and generalized hyperexponentially distributed.

1. Introduction

In this paper, we consider a modified Cramér–Lundberg risk model with constant barrier and reinvestments. Whenever the surplus in the risk model reaches barrier $M > 0$, a fraction of the surplus, $M - a$, is immediately paid out as dividend. Meanwhile, if the surplus goes below 0, that is, if there occurs a ruin, an amount of money is immediately reinvested so that the level of the surplus after the ruin becomes b $(0 \leq b < a)$.

The modified risk model has been introduced and studied by Brill and Yu (2011) and Jeong, Lim, and Lee (2009). Brill and Yu (2011) derived a renewal type equation for the stationary distribution of the surplus process and obtain the exact form of the stationary distribution when the claim sizes are exponentially distributed. Under a certain cost structure, Jeong et al. (2009) obtained the long-run average cost as a function of M and a, and illustrated an example how to find the optimal values of M and a which minimize the long-run average cost.

The risk model has been studied by many researchers. For examples, the ruin probability of the classical risk model is well summarized in Klugman, Panjer, and Willmot (2004). Gerber and Shiu (1997) obtained the joint distribution of the time of ruin, the surplus immediately before ruin and the deficit at ruin. Dufresne and Gerber (1991) generalized the classical risk model by assuming that the risk process is perturbed by diffusion and obtained the ruin probabilities. Zhang and Wang (2003) obtained the Gerber–Shiu function in the same model. Li and Lu (2005) studied the Gerber–Shiu function in the risk model with two classes of risk processes. However, most works on the risk model have been focused on the ruin probability

* Corresponding author.
E-mail address: eylee@sookmyung.ac.kr (E.Y. Lee).

http://dx.doi.org/10.1016/j.jkss.2015.01.005
1226-3192/

and related characteristics such as the first passage time to the ruin and the levels of the surplus immediately before and/or after the ruin.

In the classical risk model, the surplus process stops if a ruin occurs. However, as pointed by Borch (1969), in practice, though a ruin occurs in an insurance policy, the insurance company keeps the policy operating by a reinvestment or borrowing money from other business. In this case, the surplus process continues even though a ruin occurs. Dickson and Waters (2004) considered the Cramér–Lundberg risk model, in which the amount of deficit is reinvested if the surplus ever becomes negative. Under the barrier policy, i.e. the surplus over a barrier is paid continuously as dividend, they obtained a form of the discounted value function of the coming dividends. For the same model, Kulenko and Schmidli (2008) showed that it is optimal to reinvest the amount of deficit at ruin and to pay the dividends according to the barrier policy.

However, when the payment of each dividend incurs a fixed transaction cost, the payment of the dividend at a constant rate is impossible. In this case, for the Cramér–Lundberg risk model where the reinvestment is not considered, Bai and Guo (2010) studied the impulse control and showed that if the claim sizes are exponentially distributed, it is optimal to pay immediately a constant amount of dividend whenever the surplus surpasses the barrier. Thonhauser and Albrecher (2011) also considered the same model as that of Bai and Guo (2010) and for more general utility function, they obtained the same conclusion as Bai and Gou's, and proposed a numerical scheme to obtain the optimal dividend policy.

To the authors' best knowledge, there is no result on the Cramér–Lundberg risk model where both the payment of the dividends and reinvestment incur fixed transaction costs, while, for the diffusion risk model with the transaction costs, He and Liang (2009) showed that it is optimal to pay immediately a constant amount of dividend whenever the surplus surpasses a barrier and to reinvest up to a prescribed level whenever the surplus becomes negative, which is the same control strategy of surplus as in our model. Hence, for the Cramér–Lundberg risk model with the fixed transaction costs, the control strategy of the surplus considered in our model may be a reasonable choice.

The surplus process goes on after ruins in our model. Hence, it is worth to study the stationary behavior of the surplus process such as the average level of the surplus or the proportion of time where the surplus is in a certain level in the long-run, which is the main result of the paper. The paper is organized as follows. In Section 2, we introduce how to decompose the surplus process into two right continuous Markovian regenerative processes and we give a formula representing the stationary distribution of the surplus process as the weighted sum of those of the two decomposed processes. In Section 3, we introduce how to obtain the stationary distributions of the decomposed processes by the level crossing argument. In Sections 4 and 5, we derive the expected up-crossings of a given level during a cycle in the decomposed processes, and also the expected cycle lengths. By applying the level crossing argument to these values, we obtain the stationary distribution of the decomposed processes. Then, the explicit form of the stationary distribution of the surplus process follows immediately as a result of Section 2. In Section 6, when the claim size distribution is approximated by the generalized hyperexponential (GH) distribution, we show that the stationary distribution of the surplus process can be approximated as a sum of simple functions. In Section 7, we apply the obtained results to the special cases where the claim size is exponentially distributed, Erlang distributed, and GH distributed. Some numerical results are also given.

2. Decomposition of the surplus process

The explicit form of the surplus process analyzed in this paper is as follows. Let $S(t)$, $D(t)$, and $R(t)$ be the accumulated amount of claims, the dividends, and the reinvestments until time t, respectively, and let $\{N_S(t), t \geq 0\}$, $\{N_D(t), t \geq 0\}$, and $\{N_R(t), t \geq 0\}$ be the counting processes of the occurrence of the claims, the payment of the dividends, and the reinvestments, respectively. Then, it follows for $t \geq 0$,

$$S(t) = \sum_{i=1}^{N_S(t)} S_i,$$

$$D(t) = (M - a)\, N_D(t),$$

$$R(t) = \sum_{i=1}^{N_R(t)} R_i,$$

where S_i and R_i are the amount of the ith claim and reinvestment, respectively, for $i = 1, 2, \ldots$. In this paper, we assume that the process $\{N_S(t), t \geq 0\}$ is a Poisson process with rate λ, and that the claim size S_i, $i = 1, 2, \ldots$, is identically distributed with distribution $B(\cdot)$ and independent with any other variables. We also assume that the premium rate is a constant c. Let m be the mean of a claim size. If we define $\{X(t), t \geq 0\}$ as the surplus process, then we have, for $t \geq 0$,

$$X(t) = X(0) + ct - S(t) - D(t) + R(t). \tag{1}$$

The surplus process $\{X(t), t \geq 0\}$ is a Markovian regenerative process. The time epochs of ruins form the regeneration points, i.e. a cycle of $\{X(t), t \geq 0\}$ starts just after a ruin and ends at the next ruin. We assume that the process $\{X(t), t \geq 0\}$ is right continuous except at the time epochs of ruins. If a ruin occurs by a claim of size Y at a time t, then $X(t) = X(t-) - Y$ and the reinvestment makes $X(t+) = b$. Since the process $\{X(t), t \geq 0\}$ is right continuous except the epochs of ruins, $X(t) = a$ if the surplus reaches the barrier M at time t, i.e. $X(t-) = M$. Fig. 1 shows a sample path of the surplus process.