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Construction of main effects plans orthogonal through the block factor based on level permutation

Xue-Ping Chen [a,b], Jin-Guan Lin [a,*], Xing-Fang Huang [a]

[a] Department of Mathematics, Southeast University, Nanjing, 211189, China

[b] Department of Mathematics, Jiangsu University of Technology, Changzhou, 213001, China

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ABSTRACT

In this paper, We first demonstrate that the orthogonal property of main effects plans orthogonal through the block factor (Bagchi, 2010) remains unchanged under level permutation. However, level permutation of factors could alter their geometrical structures and statistical properties. Hence uniformity is used to further distinguish main effects plans orthogonal through the block factor (POTB). A modified optimization algorithm is proposed to search uniform or nearly uniform POTBs and many new optimal POTBs with lower-discrepancy are obtained.

1. Introduction

Main effects plans (MEP) occupy an important position in many industrial experiments when interest lies only in the main effects, assuming that all interactions between factors are negligible. An orthogonal MEP (OMEP) permits the estimation of all main effects of a factorial arrangement without correlation. Main-effect contrasts of a factor can be estimated directly in an OMEP. Moreover, the sum of squares for a factor can be simply calculated without adjustment for any other factors. An orthogonal array provides the most desirable MEP in terms of simplicity as well as precision. Two factors, F_1 and F_2 (with s_1 and s_2 levels, respectively), of an MEP D with n runs are said to be orthogonal (to each other) if they satisfy the proportional frequency condition (PFC) of Addelman (1962), which is stated as follows:

PFC: Let F_1 and F_2 be two factors in an MEP D. For every $i = 0, 1, \ldots, s_1 - 1$ and every $j = 0, 1, \ldots, s_2 - 1$, the number of runs in which factor F_1 is at level i and factor F_2 is at level j is proportional to the product of the frequencies of level i of F_1 and level j of F_2. An MEP D is said to be an OMEP if any two of its factors are orthogonal. OMEPs with each factor having equal frequency are usually derived from the widely known orthogonal arrays.

It is known that the only problem with an OMEP is that the plan often requires a large number of runs, particularly for mixed-level plans. Thus, many alternative approaches can be found in the literature. Such as "nearly orthogonal" arrays (e.g., Ma, Fang, & Liski, 2000; Wang & Wu, 1992), MEPs with blocks (e.g., Mukerjee, Dey, & Chatterjee, 2002), MEPs in which the treatment factors are pairwise orthogonal through the block factor (Bagchi, 2010; Bose & Bagchi, 2007). Among them, Bagchi (2010) obtained saturated plans orthogonal through the block factor (POTB) for a $s^{3m}2^{3m}$ experiment. Herein, $s^{3m}2^{3m}$

* Correspondence to: Department of Mathematics, Southeast University, Nanjing, 210096, China.
E-mail address: jglin@seu.edu.cn (J.-G. Lin).

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Table 1
D_1 is the three-level part of a $3^3 2^3$ POTB from Bagchi (2010) and D_2 is obtained by permutations based on D_1.

A dummy block factor	D_1			D_2		
	F_1	F_2	F_3	F_1	F_2	F_3
Batch 1	0	0	0	0	0	1
	0	1	1	0	1	0
	1	0	1	2	0	0
	1	1	0	2	1	1
Batch 2	1	1	1	2	1	0
	1	2	2	2	2	2
	2	1	2	1	1	2
	2	2	1	1	2	0
Batch 3	2	2	2	1	2	2
	2	0	0	1	0	1
	0	2	0	0	2	1
	0	0	2	0	0	2

indicates that the plan contains $3m$ factors of s levels and $3m$ factors of two levels. Moreover, it has sm blocks of size 4 each. Note that the run size of a POTB is much smaller than the one of an orthogonal MEP. In such plans, the s-level factors are nonorthogonal to the block factor but are pairwise orthogonal through the block factor. The two-level factors are orthogonal to the block factor. A construction method for a saturated connected POTB D for a $3^{3m}2^{3m}$ experiment in $3m$ blocks of size 4 each and the corresponding ANOVA table for data analysis can be found both in Bagchi (2010).

The article proceeds as follows. Section 2 first gives an example to illustrate our motivation and then introduces some basic concepts and notations of POTBs. Section 3 establishes a relationship among POTBs via level permutations. Construction via optimization algorithms for both $s^{3m}2^{3m}$ and $s^{\tilde{m}}2^{3m}(\tilde{m} < 3m)$ POTBs are discussed in Section 4. By permutation levels of existing POTBs, we obtain many new low-discrepancy designs which outperform those constructed in Bagchi (2010). Conclusions will be drawn in Section 5.

2. Preliminary

2.1. Motivation

For example, consider an experiment to study the thinning effect in Bagchi (2010). Five factors are considered: concentration of lubricants (F_1), thickness of lubrication (F_2), thickness of the door (F_3), punch speeds (F_4), and intensities of force (F_5) on the outer portions of the panel. Factors F_1, F_2 and F_3 have three levels, whereas F_4 and F_5 have two levels. Bagchi (2010) used a $3^3 2^3$ POTB with 12 runs in blocks of size 4 each. It can be constructed as follows. Let $\mathbf{H}_m$ be an orthogonal array with m rows, $m-1$ columns, two symbols, and strength 2. Such as,

$$\mathbf{H}_4 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}.$$

Then let $\mathbf{B}_l = [\mathbf{H}_4 \oplus l \pmod 3\ \mathbf{H}_4]$, $l = 0, 1, 2$. Matrix $\mathbf{H}_4 \oplus l \pmod 3$ denotes the array obtained by adding $l \pmod 3$ to each entry of $\mathbf{H}_4$. Hence, plan $(\mathbf{B}_1' \, \mathbf{B}_2' \, \mathbf{B}_3')'$ is a $3^3 2^3$ POTB, whose three-level columns (denoted by D_1) are listed in Table 1. Design D_2 in Table 1 is obtained by level permutations of factor F_1 (map (0, 1, 2) to (0, 2, 1)) and factor F_3 (map (0, 1, 2) to (1, 0, 2)). It can be verified that both designs satisfy condition (1) in Section 2 and therefore are POTBs. However, their geometrical structures could be different by level permutation. Design D_1 contains the center point with all ones, while D_2 does not.

It is well known that geometrically nonisomorphic designs have different statistical properties due to their different geometrical structures (Cheng & Wu, 2001; Cheng & Ye, 2004). To further distinguish geometrically nonisomorphic designs, uniformity is one of the mostly used criteria to compare the performance of geometrically nonisomorphic designs. More recently, Tang and Xu (2013), Tang, Xu, and Lin (2012) and Xu, Zhang, and Tang (2014) also used uniformity to compare fractional factorial designs via level permutations.

Motivated by the ideas explicitly exhibited in the aforementioned papers, we combine the ideas of level permutation and POTB and propose to construct uniform POTB via level permutations. Since there is no difference by permutations for two-level designs. Hence, in this paper, we mainly focus on the high-level part of POTBs.

2.2. POTB

In this subsection, We give some notations and background. Following Bagchi (2010), consider a plan D for an experiment with factors $F_1, F_2, \ldots,$ (possibly including a block factor L) at s-level on N runs. Let an $N \times s$ matrix $\mathbf{X}_{F_1}$ denote the incidence