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On a perturbed Sparre Andersen risk model with dividend barrier and dependence

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ABSTRACT

In this paper, we consider a Sparre Andersen risk model perturbed by a Brownian motion, where the inter-claim time and individual claim size follow some bivariate distribution. Assume that a barrier dividend strategy is applied to the surplus process, so that dividends are paid out whenever the surplus level attains a barrier b. Integral equations and integro-differential equations satisfied by the Gerber–Shiu discounted penalty functions and the expected discounted dividend payments are derived, and solutions are also given for some special cases.

1. Introduction

Consider the following Sparre Andersen risk model that is perturbed by a Brownian motion,

$$U_t = u + ct - \sum_{n=1}^{N_t} Y_n + \sigma B_t, \quad t \geq 0, \tag{1.1}$$

where $u \geq 0$ is the initial surplus and $c > 0$ is the premium rate. The claim number process $\{N_t, t \geq 0\}$ is a renewal process defined by $N_t = \max\{n : V_1 + V_2 + \cdots + V_n \leq t\}$, where V_1 is the time until the first claim arrival, and for $n \geq 2$, V_n denotes the inter-claim time between the $(n-1)$th and the nth claim arrival. $\{Y_n, n \geq 1\}$ is a sequence of strictly positive random variables representing the individual claim amounts. Finally, $\{B_t, t \geq 0\}$, independent of other processes, is a standard Brownian motion, and $\sigma > 0$ is the diffusion volatility.

Gerber (1970) first considered the compound Poisson risk model perturbed by diffusion. Since then, a lot of contributions to this perturbed risk model have been made by actuarial researchers (see Dufresne & Gerber, 1991, Gerber & Landry, 1998, Tsai, 2001, 2003, Tsai & Willmot, 2002a,b and references therein). Among these results, most attention is paid to the case when the inter-claim times and individual claim sizes are independent. Although the independence assumption can simplify the study, it is very restrictive in some applications. For example, in modeling natural catastrophic events (such as an earthquake), the intensity of the catastrophe and the time elapsed since the last catastrophe are usually dependent (see Boudreault, Cossette, Landriault, & Marceau, 2006 and Nikoloulopoulos & Karlis, 2008). Zhou and Cai (2009) consider a perturbed risk model where the premium rates are dependent on the claim sizes. Recently, Zhang and Yang (2011) add a

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Brownian motion to the compound Poisson risk model and apply the Farlie–Gumbel–Morgenstern (FGM) copula to model the dependence structure between inter-claim times and individual claim sizes. Zhang, Yang, and Yang (2012) consider a more general risk model, in which the inter-claim times and individual claim sizes follow some bivariate distribution. On the other hand, some researchers have studied the asymptotic ruin probabilities in the risk models with various dependence structures. See e.g. Li (2012), Li, Tang, and Wu (2010), Yang, Leipus, and Šiaulys (2012), Yang, Lin, and Gao (2013) and Yang, Wang, and Konstantinides (in press).

In this paper, we assume that the inter-claim times and individual claim amounts are dependent but the bivariate random vectors $\{(V_n, Y_n), n \geq 1\}$ form an i.i.d. sequence that are distributed as a generic random vector (V, Y). More precisely, we employ the dependence structure used in Zhang et al. (2012). Denote by $f_{V,Y}(t, y)$ the joint density of (V, Y), which assumes the following form,

$$f_{V,Y}(t, y) = \sum_{i=1}^{m} e^{-\lambda_i t} f_i(y), \tag{1.2}$$

where m is a strictly positive integer, $\lambda_i > 0$ for $i = 1, 2, \ldots, m$, and f_i's are continuous functions defined on $(0, \infty)$. As long as $f_{V,Y}(t, y)$ is nonnegative, some of f_i's can take negative values. It follows from (1.2) that the marginal density of the inter-claim time is a combination of exponentials. As is remarked by Zhang et al. (2012), the dependence structure described in (1.2) includes the dependence structure of Boudreault et al. (2006) and the FGM copula structure of Cossette, Marceau, and Marri (2011) as special cases. Hence, it is useful in modeling natural catastrophic events.

Recently, risk models with barrier dividend strategy have been investigated in many papers, among which some authors consider this problem in context of risk model without dependence structure between inter-claim times and individual claim sizes. See e.g. Gao and Yin (2008), Li (2006) and Yin, Shen, and Wen (2013). Landriault (2008) considers a constant barrier strategy in the compound Poisson risk model where the distribution of the next claim size depends on the last inter-claim time. Cossette et al. (2011) study the barrier strategy in the compound Poisson risk model with dependence constructed by the FGM copula. We note that most attention on the dividend problems is paid to the strictly stationary risk models or renewal risk models without diffusion perturbation. Although Cheung and Landriault (2009) study the barrier dividend strategy in a dependent risk model with diffusion perturbation, their model is Markovian stationary (or conditionally stationary).

In this paper, we will apply the barrier dividend strategy to risk model (1.1). Under this strategy, whenever the surplus reaches a barrier of constant level $b > 0$, premium income no longer goes into the surplus but is paid out as dividends to shareholders. The original surplus process in (1.1) modified by the barrier strategy, denoted by $U^b = \{U_t^b, t \geq 0\}$, can be expressed as

$$U_t^b = U_t - (\overline{U}_t - b) \vee 0, \tag{1.3}$$

where $\overline{U}_t = \sup_{0 \leq s \leq t} U_s$ is the running supremum of the process $\{U_t, t \geq 0\}$, and $x \vee y = \max\{x, y\}$. In fact, $D_t^b := (\overline{U}_t - b) \vee 0$ is the total dividend payments up to time t.

The ruin time associated with risk model (1.3) is the first passage time of U^b below zero level, i.e.

$$\tau_b = \inf\{t \geq 0 : U_t^b \leq 0\}.$$

We introduce the following Gerber–Shiu function to study the ruin problems,

$$\Phi_b(u) = \mathbb{E}[e^{-\delta \tau_b} w(U_{\tau_b -}^b, |U_{\tau_b}^b|) \mathbf{1}_{(\tau_b < \infty)} | U_0^b = u], \tag{1.4}$$

where $\delta \geq 0$ is the force of interest, $\mathbf{1}_{(\cdot)}$ is the indicator function, and $w(x_1, x_2)$ defined on $[0, \infty) \times [0, \infty)$ is a nonnegative function of the surplus before ruin $U_{\tau_b -}^b$ and the deficit at ruin $|U_{\tau_b}^b|$. According to whether or not ruin is caused by a claim, we can decompose the Gerber–Shiu function as follows,

$$\Phi_b(u) = \Phi_{b,w}(u) + \Phi_{b,d}(u),$$

where

$$\Phi_{b,w}(u) = \mathbb{E}[e^{-\delta \tau_b} w(U_{\tau_b -}^b, |U_{\tau_b}^b|) \mathbf{1}_{(\tau_b < \infty, U_{\tau_b}^b < 0)} | U_0^b = u]$$

is the Gerber–Shiu function when ruin is caused by a claim, and

$$\begin{aligned} \Phi_{b,d}(u) &= \mathbb{E}[e^{-\delta \tau_b} w(U_{\tau_b -}^b, |U_{\tau_b}^b|) \mathbf{1}_{(\tau_b < \infty, U_{\tau_b}^b = 0)} | U_0^b = u] \\ &= w(0, 0) \mathbb{E}[e^{-\delta \tau_b} \mathbf{1}_{(\tau_b < \infty, U_{\tau_b}^b = 0)} | U_0^b = u] \end{aligned}$$

is the Gerber–Shiu function when ruin occurs due to oscillation. Without loss of generality, we assume that $w(0, 0) = 1$ throughout this paper.

Given that the initial surplus is u, the expected discounted dividend payments until ruin is defined as

$$V_b(u) = \mathbb{E}[D_b | U_0^b = u],$$

where $D_b = \int_0^{\tau_b} e^{-\delta t} dD_t^b$ is the total discounted dividend payments until ruin for a discounted rate δ.